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CHAI
Humanities-Centered AI

Transformer-based Algorithmics

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May 20, 2026

Version from May 19, 2026

Topics for Today

- Hashing with open addressing
  - Linear probing
  - Quadratic probing

Acknowledgement

- Some of the upcoming content is taken from the following lectures and corresponding exercises and has been translated and partially modified:
  - Prof. Dr. Ralf Möller: »Algorithmen und Datenstrukturen – Vorlesung«
  - Dr. Magnus Bender: »Algorithmen und Datenstrukturen – Übung«

Exercise 1 – Hashing

Insert the keys (in this order):

40	12	7	26	34	51	33

into an initially empty hash table of length 9, using $h(k) = k \bmod 9$. Use **internal collision handling** (open addressing) with:

(a) **Linear probing**: $h_i(k) = (h(k) + i) \bmod 9$
(b) **Quadratic probing**: $h_i(k) = (h(k) + i^2) \bmod 9$

Open Addressing – Reminder

Probe step $i = 0, 1, 2, \ldots$ – try slot $h_i(k)$. On collision (slot occupied), increment i and retry until an empty slot is found.
Report the **final table** and all collisions (**X**) along the way.

Exercise 1 (a) – Linear Probing: First Insertions

Step	0	1	2	3	4	5	6	7	8
$h(40) = 4$					**40**				
$h(12) = 3$				**12**	40				
$h(7) = 7$				12	40			**7**	
$h(26) = 8$				12	40			7	**26**
$h(34) = 7$				12	40			**X**	26
$h_1(34) = 8$				12	40			7	**X**
$h_2(34) = 0$	**34**			12	40			7	26

Inserting 34: slot 7 holds 7 (**X**), slot 8 holds 26 (**X**), wrap to slot 0 – empty $\Rightarrow$ place 34 at index 0.

Exercise 1 (a) – Linear Probing: Final Insertions & Result

Step	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
$h(51) = 6$	34			12	40		**51**	7	26
$h(33) = 6$	34			12	40		**X**	7	26
$h_1(33) = 7$	34			12	40		51	**X**	26
$h_2(33) = 8$	34			12	40		51	7	**X**
$h_3(33) = 0$	**X**			12	40		51	7	26
$h_4(33) = 1$	34	**33**		12	40		51	7	26

Final table:

0	1	2	3	4	5	6	7	8
34	33		12	40		51	7	26

Inserting 33 caused **4 collisions** – primary clustering at slots 6–8, 0.

Exercise 1 (b) – Quadratic Probing

Step	0	1	2	3	4	5	6	7	8
$h(40) = 4$					**40**				
$h(12) = 3$				**12**	40				
$h(7) = 7$				12	40			**7**	
$h(26) = 8$				12	40			7	**26**
$h(34) = 7$				12	40			X	26
$h_1(34) = 8$				12	40			7	X
$h_2(34) = 2$			**34**	12	40			7	26
$h(51) = 6$			34	12	40		**51**	7	26
$h(33) = 6$			34	12	40		X	7	26
$h_1(33) = 7$			34	12	40		51	X	26
$h_2(33) = 1$		**33**	34	12	40		51	7	26

Final:

0	1	2	3	4	5	6	7	8
	33	34	12	40		51	7	26

Exercise 1 – Comparison & Discussion

(a) Linear Probing

0	1	2	3	4	5	6	7	8
34	33		12	40		51	7	26

Collisions: 34: 2, 33: 4 *(6 total)*

(b) Quadratic Probing

0	1	2	3	4	5	6	7	8
	33	34	12	40		51	7	26

Collisions: 34: 2, 33: 2 *(4 total)*

Observations

- **Primary clustering** (linear): the run $6, 7, 8, 0$ extends with every key hashing into it – 33 collides 4 times.
- Quadratic offsets $1, 4, 9, \ldots$ break primary clustering and halve the cascade.
- *Caveat:* for $m = 9$, $i^2 \bmod 9 \in \{0, 1, 4, 7\}$ – not all slots reachable. $\Rightarrow$ choose m **prime**.
- Both share **secondary clustering**: same $h(k) \Rightarrow$ same probe path.

Exercise 2 – Designing a Hash Function

For three-letter month names, the following hash function is proposed:

$$h(u) \;=\; \big(\mathrm{pos}(u_1) + \mathrm{pos}(u_2)\big) \bmod m,$$

where $\mathrm{pos}(u_i)$ is the position of the i-th letter of u in the alphabet (i.e., $a = 1, b = 2, \ldots, z = 26$).

Month names: `jan`, `feb`, `mar`, `apr`, `mai`, `jun`, `jul`.

Question. Can collisions occur with this hash function? If yes, give an example.

Exercise 2 – Solution & Discussion

Yes – collisions are unavoidable. The function uses only the first *two* letters, so any two months sharing those letters collide **for every** *m*:

$$h(\texttt{jun}) = h(\texttt{jul}) = (10 + 21) \bmod m = 31 \bmod m.$$

Pre-mod values for all seven months:

u	jan	feb	mar	apr	mai	jun	jul
$\mathrm{pos}(u_1) + \mathrm{pos}(u_2)$	11	11	14	17	14	31	31

Three colliding pairs already *before* the modulo: (jan, feb), (mar, mai), (jun, jul).