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CHAI
Humanities-Centered AI

Databases

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Topics for Today

- Decomposition
- Normal forms

Acknowledgement

- Some of the upcoming content is taken from the following lecture (and corresponding exercises) and has been translated and partially modified:
  - PD Dr. habil. Özgür L. Özçep: »Datenbanken«

Decomposition I

Let the schema $\mathcal{R} = \{A, B, C, D, E\}$ be given with the functional dependencies

- $A \rightarrow BC$,
- $CD \rightarrow E$,
- $B \rightarrow D$, and
- $E \rightarrow A$.

Is $\{A\}$ a candidate key? Why (not)?

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  - $A \to A$ holds due to reflexivity

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  - Hence, we have $A \rightarrow CD$, and with $CD \rightarrow E$ also $A \rightarrow E$ by transitivity

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Is $\{A\}$ a candidate key? Why (not)? $\checkmark$

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- It remains to be shown that $\{A\} \rightarrow \mathcal{R}$
  - $A \rightarrow A$ holds due to reflexivity
  - $A \rightarrow B$, $A \rightarrow C$ are given
  - $A \rightarrow D$ follows by transitivity from $A \rightarrow B$ and $B \rightarrow D$
  - Hence, we have $A \rightarrow CD$, and with $CD \rightarrow E$ also $A \rightarrow E$ by transitivity

Decomposition II

Let the schema $\mathcal{R} = \{A, B, C, D, E\}$ be given with the functional dependencies

$$A \to BC, \quad CD \to E, \quad B \to D, \quad E \to A.$$

Let $\mathcal{R}_1 = \{A, B, C\}$ and $\mathcal{R}_2 = \{A, D, E\}$ be a decomposition of $\mathcal{R}$. Show that this decomposition is lossless (with respect to the given functional dependencies).

The decomposition $\mathcal{R} = \mathcal{R}_1 \cup \mathcal{R}_2$ is lossless if $\mathcal{R}_1 \cap \mathcal{R}_2 \to \mathcal{R}_1$ or $\mathcal{R}_1 \cap \mathcal{R}_2 \to \mathcal{R}_2$.

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- $\mathcal{R}_1 \cap \mathcal{R}_2 = \{A\}$

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- $\mathcal{R}_1 \cap \mathcal{R}_2 = \{A\}$
- As $\{A\}$ is a candidate key, it holds that $\{A\} \to \mathcal{R}$ and thus also $\{A\} \to \mathcal{R}_1$ ($\mathcal{R}_1 \cap \mathcal{R}_2 \to \mathcal{R}_1$) and $\{A\} \to \mathcal{R}_2$ ($\mathcal{R}_1 \cap \mathcal{R}_2 \to \mathcal{R}_2$)

Normal Forms – 1NF

Is the following table in 1NF? If not, transform it into 1NF.

CustomerID	Name	Items
1	Alice	{Bread, Milk}
2	Bob	{Bread, Eggs}
3	Carol	{Milk, Eggs}

Normal Forms – 1NF

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3	Carol	{Milk, Eggs}

No, 1NF requires simple domains.

Normal Forms – 1NF

Transformed into 1NF, the table looks as follows:

CustomerID	Name	Items
1	Alice	Bread
1	Alice	Milk
2	Bob	Bread
2	Bob	Eggs
3	Carol	Milk
3	Carol	Eggs

Normal Forms – 2NF

Is the following table in 2NF?

StudentID	LectureID	Name	Semester
12345	CS100	Alice	1
12321	CS100	Bob	3
12321	CS101	Bob	3
54321	CS102	Carol	2
54321	CS103	Carol	2
54321	CS104	Carol	2
54321	CS105	Carol	2

Normal Forms – 2NF

StudentID	LectureID	Name	Semester
12345	CS100	Alice	1
12321	CS100	Bob	3
12321	CS101	Bob	3
54321	CS102	Carol	2
54321	CS103	Carol	2
54321	CS104	Carol	2
54321	CS105	Carol	2

A relation $\mathcal{R}$ is in 2NF if each non-key attribute $A \in \mathcal{R}$ is fully functionally dependent on every candidate key of $\mathcal{R}$.

Normal Forms – 2NF

{StudentID, LectureID} is a candidate key, but

- {StudentID, LectureID} $\rightarrow^{\bullet}$ {Name} and
- {StudentID, LectureID} $\rightarrow^{\bullet}$ {Semester}

do not hold because we have

- {StudentID} $\rightarrow$ {Name} and
- {StudentID} $\rightarrow$ {Semester}.

Thus, the table is not in 2NF.

StudentID	LectureID	Name	Semester
12345	CS100	Alice	1
12321	CS100	Bob	3
12321	CS101	Bob	3
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54321	CS103	Carol	2
54321	CS104	Carol	2
54321	CS105	Carol	2

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Normal Forms – Boyce-Codd Normal Form (BCNF)

A relation $\mathcal{R}$ is in BCNF if

(1) $\mathcal{R}$ is in 2NF (i.e., each non-key attribute $A \in \mathcal{R}$ is fully functionally dependent on every candidate key of $\mathcal{R}$), and

(2) for each functional dependency $\alpha \rightarrow \beta$ that holds in $\mathcal{R}$, it holds that

   (a) $\beta \subseteq \alpha$ (i.e., the dependency is trivial), or

   (b) α is a super key of $\mathcal{R}$.

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Let $\mathcal{R} = \{A, B, C, D\}$ with functional dependencies

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- It holds that $\{A, B\} \to^{\bullet} D$ and $\{A, C\} \to^{\bullet} D$

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Is $\mathcal{R}$ in 2NF? Yes.

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