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CHAI
Humanities-Centered AI

Databases

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Topics for Today

- Recap: Functional dependencies
- Keys
- Relational algebra
- Attribute closure

Acknowledgement

- Some of the upcoming content is taken from the following lecture (and corresponding exercises) and has been translated and partially modified:
  - PD Dr. habil. Özgür L. Özçep: »Datenbanken«

Recap: Functional Dependencies

- Schema: $\mathcal{R} = \{A, B, C, D\}$
- Instance R:

A	B	C	D
a_4	b_2	c_4	d_3
a_1	b_1	c_1	d_1
a_1	b_1	c_1	d_2
a_2	b_2	c_3	d_2
a_3	b_2	c_4	d_3

Let $\alpha \subseteq \mathcal{R}$, $\beta \subseteq \mathcal{R}$. It holds that $\alpha \rightarrow \beta$ if and only if $\forall r, s \in R$: $r.\alpha = s.\alpha \Rightarrow r.\beta = s.\beta$.

Which of the following functional dependencies hold?

- $\{A\} \rightarrow \{B\}$
- $\{C, D\} \rightarrow \{A\}$
- $\{C, D\} \rightarrow \{B\}$
- $\{B\} \rightarrow \{C\}$
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- $\{C, D\} \to \{B\}$ ✓
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- $\{B\} \to \{A\}$ ✗

Keys – Comprehension Questions I

Are the following statements correct?

- »If $\{A\} \rightarrow \{B\}$ holds, then $\{B\} \rightarrow \{A\}$ also holds.«
- »For a super key α, it holds that $\alpha \rightarrow^{\bullet} \mathcal{R}$.«

Keys – Comprehension Questions I

Are the following statements correct?

- »If $\{A\} \rightarrow \{B\}$ holds, then $\{B\} \rightarrow \{A\}$ also holds.«
  - Incorrect. Counterexample: $R(A, B) = \{(a_1, b_1), (a_2, b_1)\}$.
- »For a super key α, it holds that $\alpha \rightarrow^{\bullet} \mathcal{R}$.«

Keys – Comprehension Questions I

Are the following statements correct?

- »If $\{A\} \rightarrow \{B\}$ holds, then $\{B\} \rightarrow \{A\}$ also holds.«
  - Incorrect. Counterexample: $R(A, B) = \{(a_1, b_1), (a_2, b_1)\}$.
- »For a super key α, it holds that $\alpha \rightarrow^{\bullet} \mathcal{R}$.«
  - Incorrect. Although a super key determines all attributes, it is not necessarily minimal.

Keys – Comprehension Questions II

Are the following statements correct?

- »For a candidate key α, there exists no candidate key γ with $\gamma \subsetneq \alpha$.«
- »For a candidate key α, there exists no candidate key γ with $\alpha \subsetneq \gamma$.«
- »For a candidate key α, there exists no candidate key β with $|\beta| < |\alpha|$.«

Keys – Comprehension Questions II

Are the following statements correct?

- »For a candidate key α, there exists no candidate key γ with $\gamma \subsetneq \alpha$.«
  - Correct. This holds due to the minimality of α with regard to set inclusion.
- »For a candidate key α, there exists no candidate key γ with $\alpha \subsetneq \gamma$.«
- »For a candidate key α, there exists no candidate key β with $|\beta| < |\alpha|$.«

Keys – Comprehension Questions II

Are the following statements correct?

- »For a candidate key α, there exists no candidate key γ with $\gamma \subsetneq \alpha$.«
  - Correct. This holds due to the minimality of α with regard to set inclusion.
- »For a candidate key α, there exists no candidate key γ with $\alpha \subsetneq \gamma$.«
  - Correct. Otherwise, the minimality of γ would be violated.
- »For a candidate key α, there exists no candidate key β with $|\beta| < |\alpha|$.«

Keys – Comprehension Questions II

Are the following statements correct?

- »For a candidate key α, there exists no candidate key γ with $\gamma \subsetneq \alpha$.«
  - Correct. This holds due to the minimality of α with regard to set inclusion.
- »For a candidate key α, there exists no candidate key γ with $\alpha \subsetneq \gamma$.«
  - Correct. Otherwise, the minimality of γ would be violated.
- »For a candidate key α, there exists no candidate key β with $|\beta| < |\alpha|$.«
  - Incorrect. Minimality refers only to set inclusion, not to the cardinality of the attribute sets. Counterexample: $R(A, B, C) = \{(a_1, b_1, c_1), (a_1, b_2, c_2), (a_2, b_1, c_3)\}$. Both $A \rightarrow C$ and $B \rightarrow C$ do not hold, while $A, B \rightarrow C$ holds. It also holds that $C \rightarrow \mathcal{R}$.

Relational Algebra – Operators

Given the following relations R and S, specify the union ($R \cup S$), intersection ($R \cap S$), and difference ($R \setminus S$) of the two relations.

R:

ProjectID	Title	Budget
1	Project A	300.000
2	Project B	200.000
3	Project C	200.000

S:

ProjectID	Title	Budget
2	Project B	200.000
3	Project C	200.000
4	Project D	100.000

Relational Algebra – Operators

R:

ProjectID	Title	Budget
1	Project A	300.000
2	Project B	200.000
3	Project C	200.000

S:

ProjectID	Title	Budget
2	Project B	200.000
3	Project C	200.000
4	Project D	100.000

$R \cup S$:

ProjectID	Title	Budget
1	Project A	300.000
2	Project B	200.000
3	Project C	200.000
4	Project D	100.000

Relational Algebra – Operators

R:

ProjectID	Title	Budget
1	Project A	300.000
2	Project B	200.000
3	Project C	200.000

S:

ProjectID	Title	Budget
2	Project B	200.000
3	Project C	200.000
4	Project D	100.000

$R \cap S$:

ProjectID	Title	Budget
2	Project B	200.000
3	Project C	200.000

Relational Algebra – Operators

R:

ProjectID	Title	Budget
1	Project A	300.000
2	Project B	200.000
3	Project C	200.000

S:

ProjectID	Title	Budget
2	Project B	200.000
3	Project C	200.000
4	Project D	100.000

$R \setminus S$:

ProjectID	Title	Budget
1	Project A	300.000

Relational Algebra – Operators

Given the following relations R and S, specify the Cartesian product $R \times S$ and the join $R \bowtie_{Nr1=Nr2} S$ of the two relations.

R:

Nr1	Title	Budget
1	Project A	300.000
2	Project B	200.000
3	Project C	200.000

S:

Nr2	Note
1	ABC
2	XYZ
3	ABC

Relational Algebra – Operators

R:

Nr1	Title	Budget
1	Project A	300.000
2	Project B	200.000
3	Project C	200.000

S:

Nr2	Note
1	ABC
2	XYZ
3	ABC

$R \times S$:

Nr1	Title	Budget	Nr2	Note
1	Project A	300.000	1	ABC
1	Project A	300.000	2	XYZ
1	Project A	300.000	3	ABC
2	Project B	200.000	1	ABC
2	Project B	200.000	2	XYZ
2	Project B	200.000	3	ABC
3	Project C	200.000	1	ABC
3	Project C	200.000	2	XYZ
3	Project C	200.000	3	ABC

Relational Algebra – Operators

R:

Nr1	Title	Budget
1	Project A	300.000
2	Project B	200.000
3	Project C	200.000

S:

Nr2	Note
1	ABC
2	XYZ
3	ABC

$R \bowtie_{Nr1=Nr2} S$:

Nr1	Title	Budget	Note
1	Project A	300.000	ABC
2	Project B	200.000	XYZ
3	Project C	200.000	ABC

Relational Algebra – Operators

Given the following relation R, specify the selection $\sigma_{Budget>200.000}(R)$ and the projection $\pi_{(Title,Budget)}(R)$.

R:

Nr	Title	Budget
1	Project A	300.000
2	Project B	200.000
3	Project C	200.000

Relational Algebra – Operators

Given the following relation R, specify the selection $\sigma_{Budget>200.000}(R)$ and the projection $\pi_{(Title,Budget)}(R)$.

R:

Nr	Title	Budget
1	Project A	300.000
2	Project B	200.000
3	Project C	200.000

$\sigma_{Budget>200.000}(R)$:

Nr	Title	Budget
1	Project A	300.000

Relational Algebra – Operators

Given the following relation R, specify the selection $\sigma_{Budget>200.000}(R)$ and the projection $\pi_{(Title,Budget)}(R)$.

R:

Nr	Title	Budget
1	Project A	300.000
2	Project B	200.000
3	Project C	200.000

$\sigma_{Budget>200.000}(R)$:

Nr	Title	Budget
1	Project A	300.000

$\pi_{(Title,Budget)}(R)$:

Title	Budget
Project A	300.000
Project B	200.000
Project C	200.000

Relational Algebra – Comprehension Questions

Which of the following equivalences hold?

- $\sigma_{p_1}(\sigma_{p_2}(R)) = \sigma_{(p_1 \wedge p_2)}(R)$
- $\sigma_p(\sigma_q(R)) = \sigma_q(\sigma_p(R))$
- $R_1 \cup (R_2 \cap R_3) = (R_1 \cap R_2) \cup (R_1 \cap R_3)$
- $\sigma_{p_1}(\sigma_{p_2}(R)) = \sigma_{(p_1 \vee p_2)}(R)$
- $R_1 \cap (R_2 \cup R_3) = (R_1 \cap R_2) \cup (R_1 \cap R_3)$
- $R_1 \bowtie R_2 = R_2 \bowtie R_1$

Relational Algebra – Comprehension Questions

Which of the following equivalences hold?

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Relational Algebra – Comprehension Questions

Which of the following equivalences hold?

- $\sigma_{p_1}(\sigma_{p_2}(R)) = \sigma_{(p_1 \wedge p_2)}(R)$
  - Correct:

$$\begin{aligned}
\sigma_{(p_1 \wedge p_2)}(R) &= \{r \in R \mid r \models p_1 \wedge p_2\} \\
&= \{r \in R \mid r \models p_1 \text{ and } r \models p_2\} \\
&= \{r \in R \mid r \models p_1\} \cap \{r \in R \mid r \models p_2\} \\
&= \{r \in \{r' \in R \mid r' \models p_1\} \mid r \models p_2\} \\
&= \sigma_{p_1}(\sigma_{p_2}(R)).
\end{aligned}$$

Relational Algebra – Comprehension Questions

Which of the following equivalences hold?

- $\sigma_p(\sigma_q(R)) = \sigma_q(\sigma_p(R))$

Relational Algebra – Comprehension Questions

Which of the following equivalences hold?

- $\sigma_p(\sigma_q(R)) = \sigma_q(\sigma_p(R))$
  - Correct:

$$\begin{aligned}\sigma_p(\sigma_q(R)) &= \sigma_{(p \wedge q)}(R) && \text{(shown in the previous equivalence statement)}\\ &= \sigma_{(q \wedge p)}(R)\\ &= \sigma_q(\sigma_p(R)).\end{aligned}$$

Relational Algebra – Comprehension Questions

Which of the following equivalences hold?

- $R_1 \cup (R_2 \cap R_3) = (R_1 \cap R_2) \cup (R_1 \cap R_3)$

Relational Algebra – Comprehension Questions

Which of the following equivalences hold?

- $R_1 \cup (R_2 \cap R_3) = (R_1 \cap R_2) \cup (R_1 \cap R_3)$
  - Incorrect. Counterexample: Let $R_1 = \{r_1, r_2\}, R_2 = \{r_2, r_3\}, R_3 = \{r_1, r_3\}$.

$$\begin{aligned} R_1 \cup (R_2 \cap R_3) &= \{r_1, r_2\} \cup (\{r_2, r_3\} \cap \{r_1, r_3\}) \\ &= \{r_1, r_2\} \cup \{r_3\} \\ &= \{r_1, r_2, r_3\} \\ &\neq \{r_1, r_2\} \\ &= \{r_2\} \cup \{r_1\} \\ &= (\{r_1, r_2\} \cap \{r_2, r_3\}) \cup (\{r_1, r_2\} \cap \{r_1, r_3\}) \\ &= (R_1 \cap R_2) \cup (R_1 \cap R_3). \end{aligned}$$

Relational Algebra – Comprehension Questions

Which of the following equivalences hold?

- $\sigma_{p_1}(\sigma_{p_2}(R)) = \sigma_{(p_1 \vee p_2)}(R)$

Relational Algebra – Comprehension Questions

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- $\sigma_{p_1}(\sigma_{p_2}(R)) = \sigma_{(p_1 \vee p_2)}(R)$
  - Incorrect. Counterexample: Let $R = \{a_1, b_1\}$ with schema $\mathcal{R} = \{A, B\}$.

$$
\begin{aligned}
\sigma_{A=a_1}(\sigma_{B=b_1}(R)) &= \sigma_{A=a_1}(\sigma_{B=b_1}(\{a_1, b_1\})) \\
&= \sigma_{A=a_1}(\{b_1\}) \\
&= \emptyset \\
&\neq \{a_1, b_1\} \\
&= \sigma_{(A=a_1 \vee B=b_1)}(\{a_1, b_1\}) \\
&= \sigma_{(A=a_1 \vee B=b_1)}(R).
\end{aligned}
$$

Relational Algebra – Comprehension Questions

Which of the following equivalences hold?

- $R_1 \cap (R_2 \cup R_3) = (R_1 \cap R_2) \cup (R_1 \cap R_3)$

Relational Algebra – Comprehension Questions

Which of the following equivalences hold?

- $R_1 \cap (R_2 \cup R_3) = (R_1 \cap R_2) \cup (R_1 \cap R_3)$
  - Correct. This follows directly from the distributive law of $\cap$ over $\cup$ (stemming from the distributive law of $\wedge$ over $\vee$).

Relational Algebra – Comprehension Questions

Which of the following equivalences hold?

- $R_1 \bowtie R_2 = R_2 \bowtie R_1$

Relational Algebra – Comprehension Questions

Which of the following equivalences hold?

- $R_1 \bowtie R_2 = R_2 \bowtie R_1$
  - Incorrect. Counterexample: Let $R_1 = \{r_1\}$ and $R_2 = \{r_2\}$.

$$
\begin{aligned}
R_1 \bowtie R_2 &= \{(r_1, r_2)\} \\
&\neq \{(r_2, r_1)\} \\
&= R_2 \bowtie R_1 .
\end{aligned}
$$

Attribute Closure

Let the following functional dependencies be given:

- Functional dependencies: $F = \{A \rightarrow B, B \rightarrow C, C \rightarrow AD\}$

Determine the attribute closure α^+ for the attribute set $\alpha = \{A\}$.

```
attributeClosure(F, α)
  result := α; result' := ∅
  while result' ≠ result do
    result' := result
    foreach β → γ ∈ F do
      if β ⊆ result then
        result := result ∪ γ
  return result
```

Attribute Closure

(1) Initialize: result := $\{A\}$, result' := $\{\}$

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- $F = \{A \rightarrow B, B \rightarrow C, C \rightarrow AD\}$
- $\alpha = \{A\}$

Attribute Closure

(1) Initialize: result := $\{A\}$, result' := $\{\}$
(2) result' = $\{\} \neq$ result = $\{A\}$:
   (a) result' := $\{A\}$
   (b) $A \to B$: result := result $\cup \{B\}$

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attributeClosure(F, α)
  result := α; result' := ∅
  while result' ≠ result do
    result' := result
    foreach β → γ ∈ F do
      if β ⊆ result then
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- $F = \{A \to B, B \to C, C \to AD\}$
- $\alpha = \{A\}$

Attribute Closure

(1) Initialize: result := $\{A\}$, result' := $\{\}$
(2) result' = $\{\} \neq$ result = $\{A\}$:
   (a) result' := $\{A\}$
   (b) $A \rightarrow B$: result := result $\cup \{B\}$
(3) result' = $\{A\} \neq$ result = $\{A, B\}$:
   (a) result' := $\{A, B\}$
   (b) $B \rightarrow C$: result := result $\cup \{C\}$

```
attributeClosure(F, α)
  result := α; result' := ∅
  while result' ≠ result do
    result' := result
    foreach β → γ ∈ F do
      if β ⊆ result then
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(2) result' = $\{\} \neq$ result = $\{A\}$:
   (a) result' := $\{A\}$
   (b) $A \rightarrow B$: result := result $\cup \{B\}$
(3) result' = $\{A\} \neq$ result = $\{A, B\}$:
   (a) result' := $\{A, B\}$
   (b) $B \rightarrow C$: result := result $\cup \{C\}$
(4) result' = $\{A, B\} \neq$ result = $\{A, B, C\}$:
   (a) result' := $\{A, B, C\}$
   (b) $C \rightarrow AD$: result := result $\cup \{A, D\}$

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attributeClosure(F, α)
  result := α; result' := ∅
  while result' ≠ result do
    result' := result
    foreach β → γ ∈ F do
      if β ⊆ result then
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- $F = \{A \rightarrow B, B \rightarrow C, C \rightarrow AD\}$
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Attribute Closure

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(2) result' = $\{\} \neq$ result = $\{A\}$:
  (a) result' := $\{A\}$
  (b) $A \rightarrow B$: result := result $\cup \{B\}$
(3) result' = $\{A\} \neq$ result = $\{A, B\}$:
  (a) result' := $\{A, B\}$
  (b) $B \rightarrow C$: result := result $\cup \{C\}$
(4) result' = $\{A, B\} \neq$ result = $\{A, B, C\}$:
  (a) result' := $\{A, B, C\}$
  (b) $C \rightarrow AD$: result := result $\cup \{A, D\}$
(5) result' = $\{A, B, C\} \neq$ result = $\{A, B, C, D\}$:
  (a) result' := $\{A, B, C, D\}$

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attributeClosure(F, α)
  result := α; result' := ∅
  while result' ≠ result do
    result' := result
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        result := result ∪ γ
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- $F = \{A \rightarrow B, B \rightarrow C, C \rightarrow AD\}$
- $\alpha = \{A\}$

Attribute Closure

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(2) result' = $\{\} \neq$ result = $\{A\}$:
   (a) result' := $\{A\}$
   (b) $A \rightarrow B$: result := result $\cup \{B\}$
(3) result' = $\{A\} \neq$ result = $\{A, B\}$:
   (a) result' := $\{A, B\}$
   (b) $B \rightarrow C$: result := result $\cup \{C\}$
(4) result' = $\{A, B\} \neq$ result = $\{A, B, C\}$:
   (a) result' := $\{A, B, C\}$
   (b) $C \rightarrow AD$: result := result $\cup \{A, D\}$
(5) result' = $\{A, B, C\} \neq$ result = $\{A, B, C, D\}$:
   (a) result' := $\{A, B, C, D\}$
(6) return result = $\{A, B, C, D\}$

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  result := α; result' := ∅
  while result' ≠ result do
    result' := result
    foreach β → γ ∈ F do
      if β ⊆ result then
        result := result ∪ γ
  return result
```

- $F = \{A \rightarrow B, B \rightarrow C, C \rightarrow AD\}$
- $\alpha = \{A\}$