UNIVERSITÄT ZU LÜBECK
INSTITUTE OF INFORMATION SYSTEMS

German
Research Center
for Artificial
Intelligence

Lifting Factor Graphs with Some Unknown Factors

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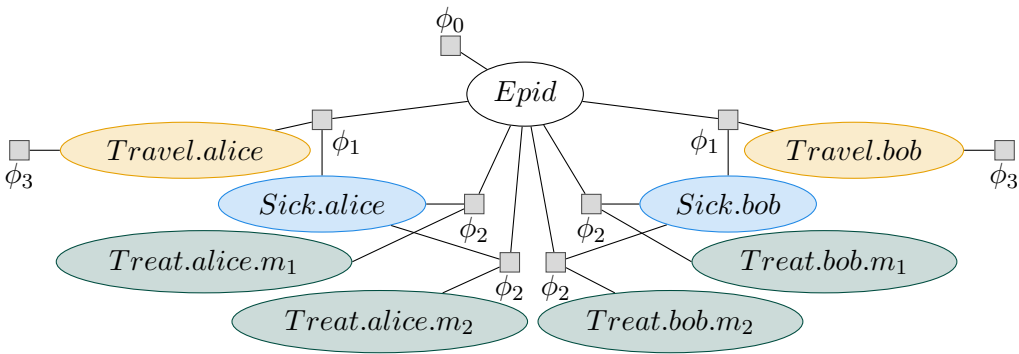
Motivation

Factor Graphs (Kschischang, Frey, and Loeliger, 2001)

- Factor graph as a compact encoding of a full joint probability distribution
- Semantics of a factor graph G over a set of factors $\mathbf{\Phi}$:

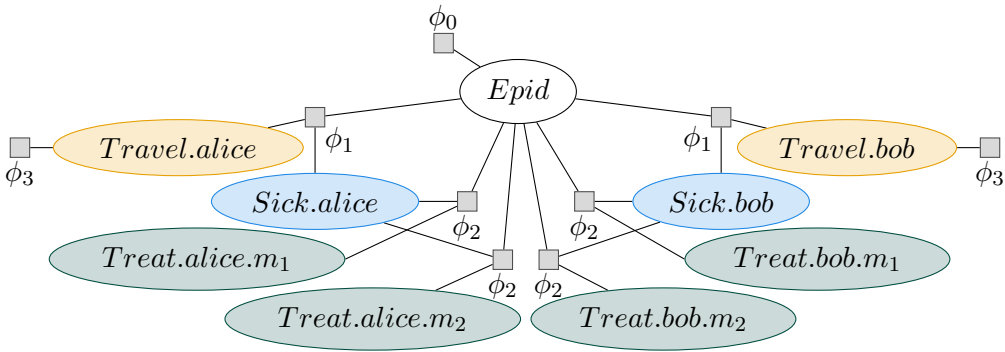
$$P_G = \frac{1}{Z} \prod_{\phi \in \mathbf{\Phi}} \phi$$



Motivation

The Problem with Propositional Factor Graphs

- Adding new people to the population increases the size of the factor graph
- Probabilistic inference requires time exponential in the number of random variables



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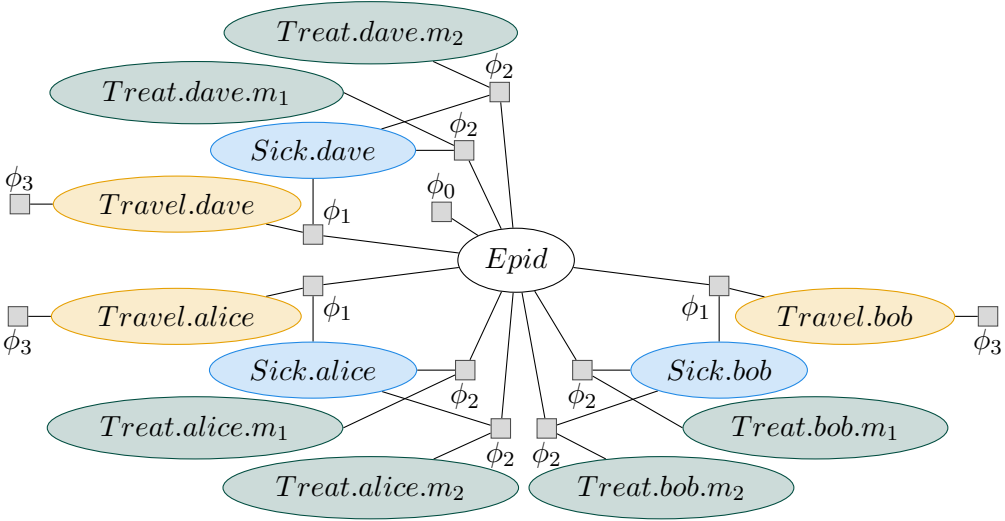

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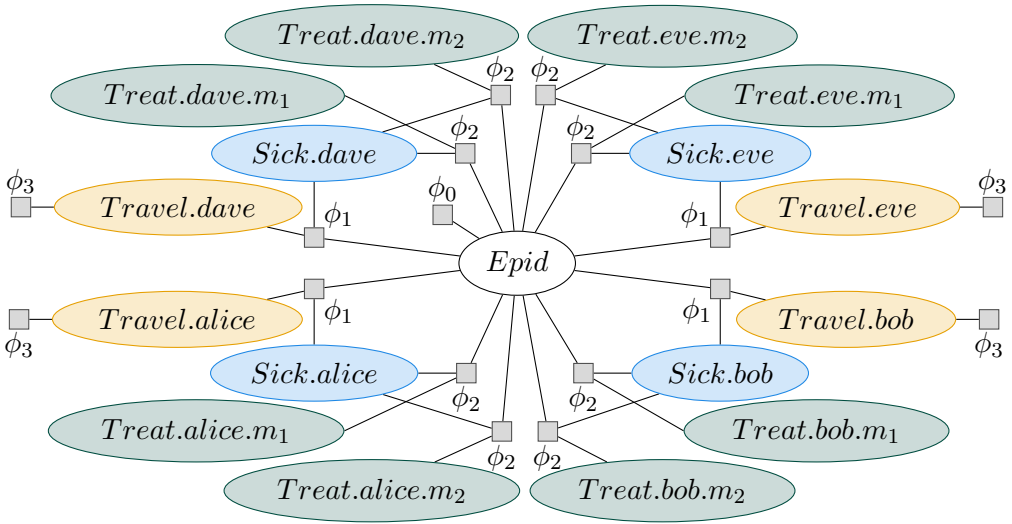
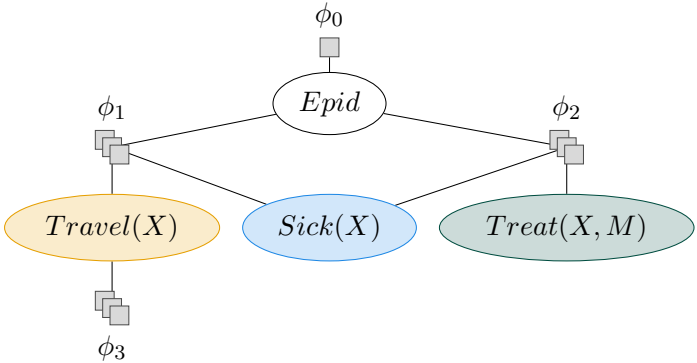


Motivation

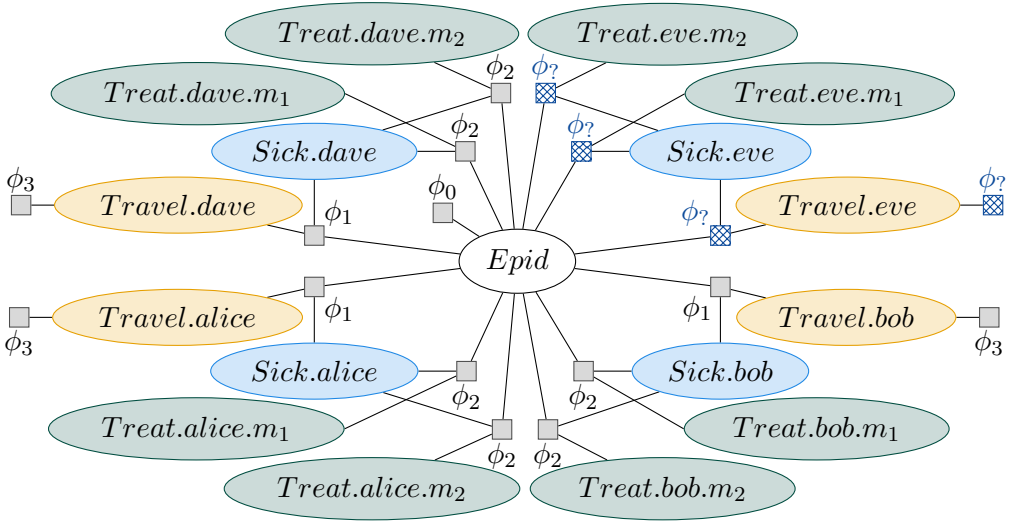
Parametric Factor Graphs (Poole, 2003)

- Assumption: Symmetries in a graph
- Introduce logical variables to represent groups of random variables
  - $\mathcal{D}(X) = \{alice, bob, dave, eve\}$
  - $\mathcal{D}(M) = \{m_1, m_2\}$



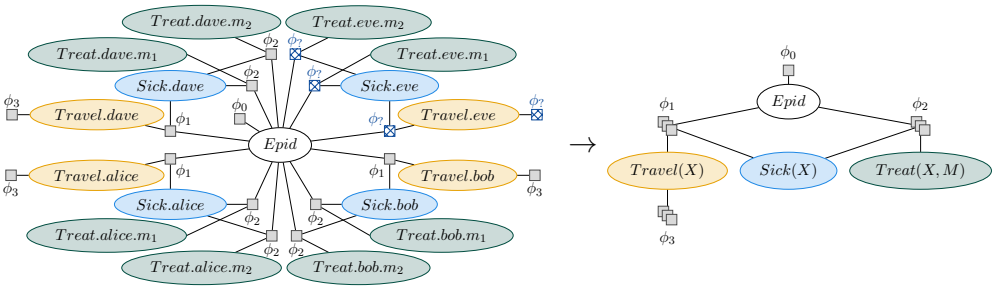
Motivation

But What to Do if Some Factors Are Unknown?



Problem Setup

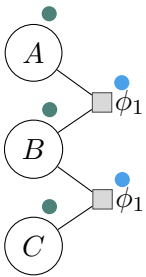
- Input: A factor graph G possibly containing unknown factors
- Output: A lifted representation of G



Prerequisite: Colour Passing Algorithm

(Ahmadi et al., 2013)

- Assign colours to random variables depending on their ranges and evidence
- Assign colour to factors depending on their potentials
- Pass colours around to detect symmetries in the graph



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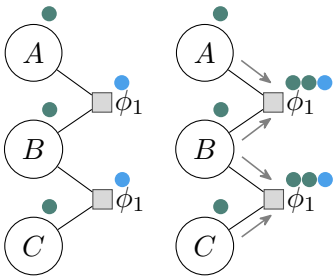

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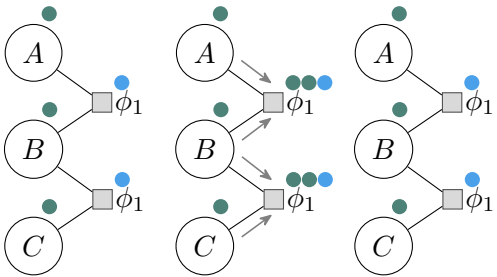

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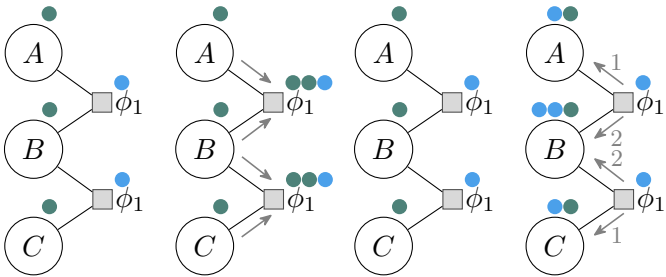

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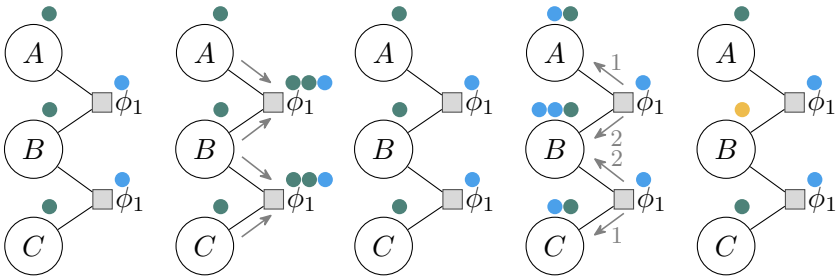

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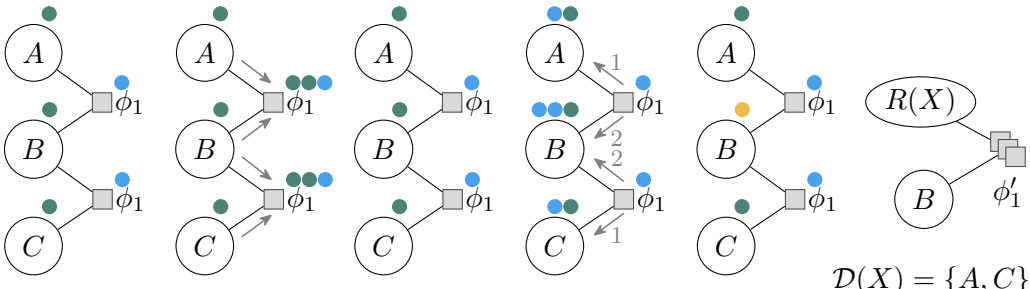
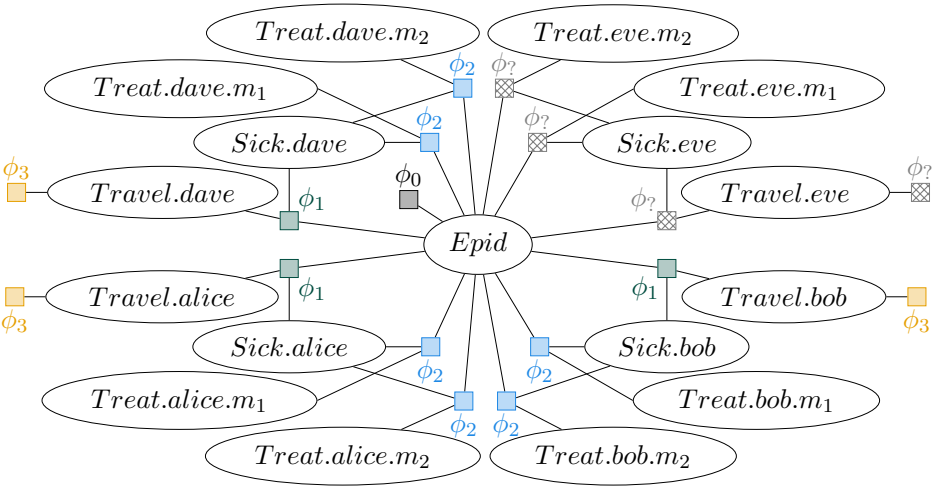


Lifting Factor Graphs with Some Unknown Factors (LIFAGU) Algorithm

- What colour to assign to the unknown factors?
  - Potentials are missing
  - Only available information: Surrounding graph structure
- General idea:
  1. Known factors are coloured according to their potentials
  2. Unknown factors are coloured according to their 2-step neighbourhood
  3. Assign unknown factors and known factors the same colour if their 2-step neighbourhoods are symmetric
  4. Run the standard colour passing algorithm

Lifting Factor Graphs with Some Unknown Factors (LIFAGU) Algorithm

1. Known factors are coloured according to their potentials



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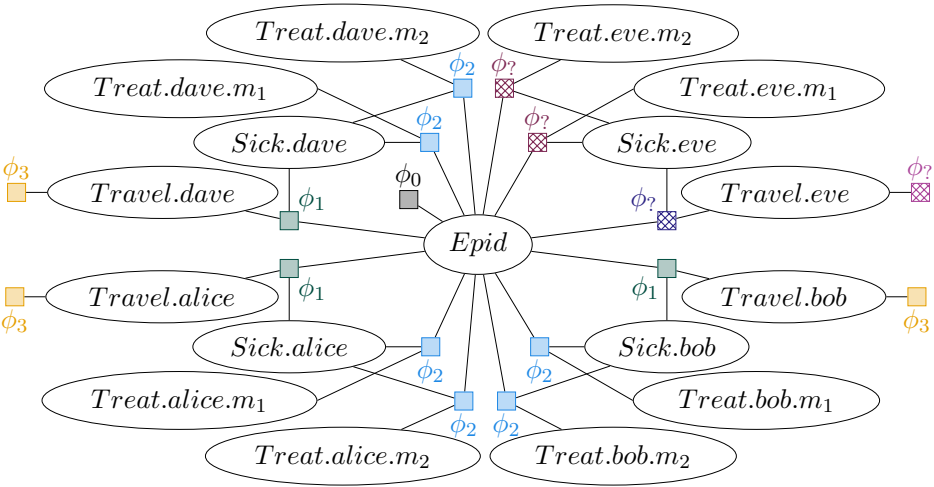
2. Unknown factors are coloured according to their 2-step neighbourhood



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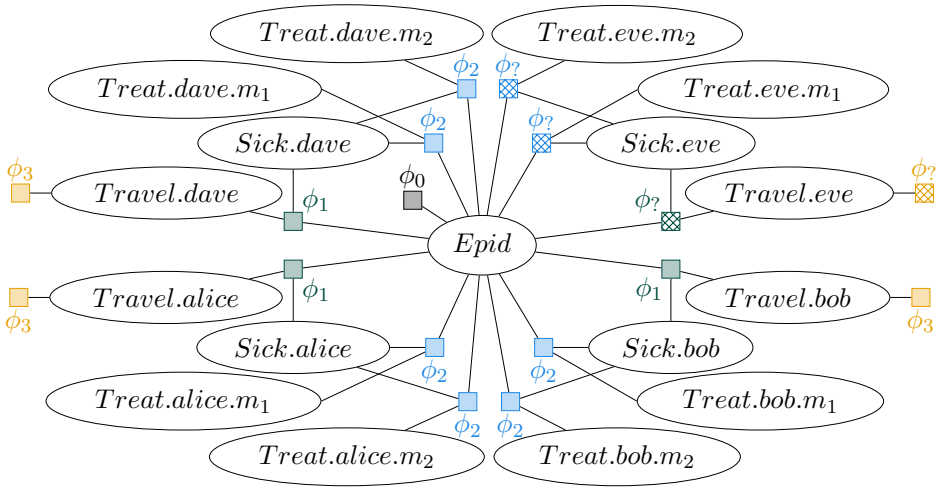

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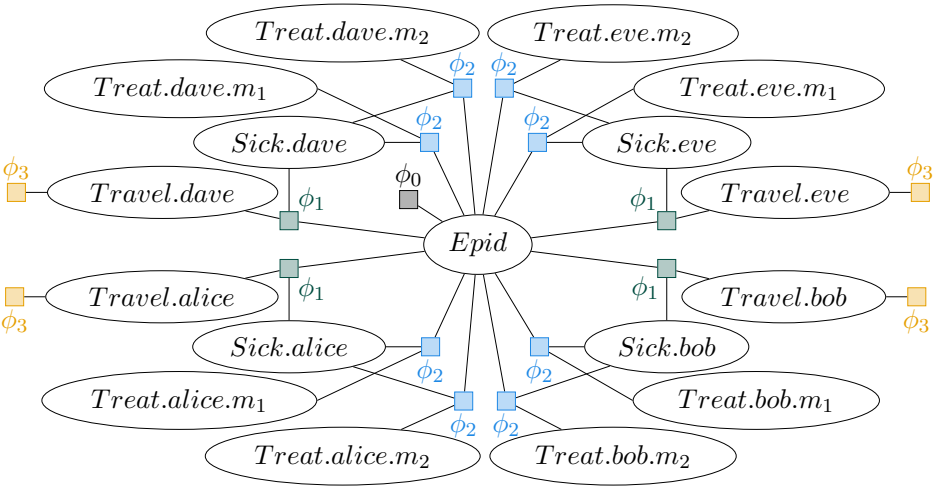
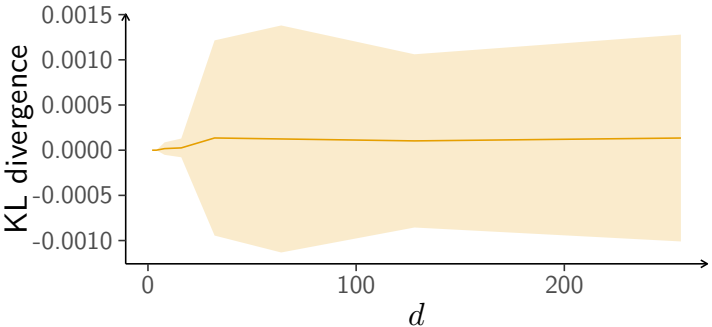


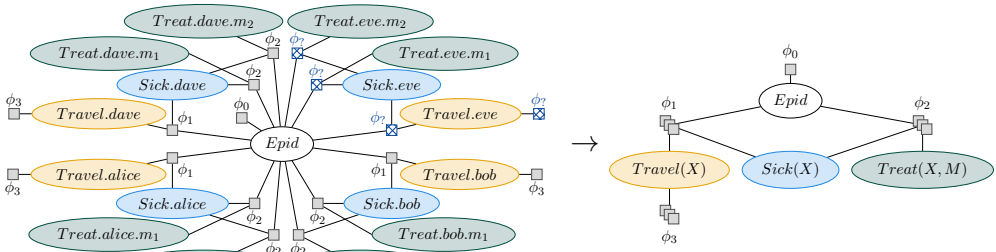
Empirical Evaluation

- Generate factor graphs where all factors are known
- Randomly remove potentials of 5-10 percent of the factors
- Run LIFAGU to obtain a lifted representation
- Perform probabilistic inference on the ground truth and the lifted representation



Conclusion

- Construct a lifted representation for factor graphs with unknown factors
- Transfer known potentials to unknown factors
- Ensure a well-defined semantics and allow for lifted inference



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- Kschischang, Frank R., Brendan J. Frey, and Hans-Andrea Loeliger (2001). »Factor Graphs and the Sum-Product Algorithm«. In: *IEEE Transactions on Information Theory* 47, pp. 498–519.
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