Monthly StarAI Update

Recent Research

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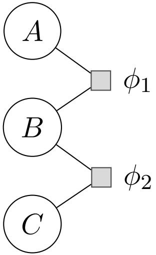
Factor Graphs

- Factor graph as a compact encoding of a full joint probability distribution
- Semantics of a factor graph G over a set of factors $\boldsymbol{\Phi}$:

$$P_G = \frac{1}{Z} \prod_{\phi \in \boldsymbol{\Phi}} \phi$$

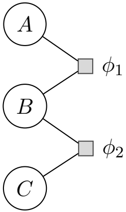


A	B	$\phi_1(A, B)$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

C	B	$\phi_2(C, B)$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

Frank R. Kschischang, Brendan J. Frey, and Hans-Andrea Loeliger (2001). »Factor Graphs and the Sum-Product Algorithm«. *IEEE Transactions on Information Theory* 47, pp. 498–519.

Symmetries in Factor Graphs



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$$P(B) = \sum_{a \in \mathcal{R}(A)} \sum_{c \in \mathcal{R}(C)} P(A = a, B, C = c)$$

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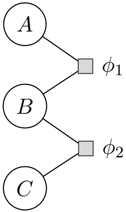

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$$
\begin{aligned}
P(B) &= \sum_{a \in \mathcal{R}(A)} \sum_{c \in \mathcal{R}(C)} P(A = a, B, C = c) \\
&= \frac{1}{Z} \cdot \sum_{a \in \mathcal{R}(A)} \sum_{c \in \mathcal{R}(C)} \phi_1(A = a, B) \cdot \phi_2(C = c, B)
\end{aligned}
$$

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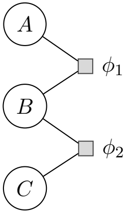
$$= \frac{1}{Z} \cdot \sum_{a \in \mathcal{R}(A)} \phi_1(A = a, B) \cdot \sum_{c \in \mathcal{R}(C)} \phi_2(C = c, B)$$



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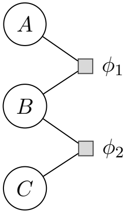
Symmetries in Factor Graphs



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\begin{aligned}
P(B) &= \sum_{a \in \mathcal{R}(A)} \sum_{c \in \mathcal{R}(C)} P(A = a, B, C = c) \\
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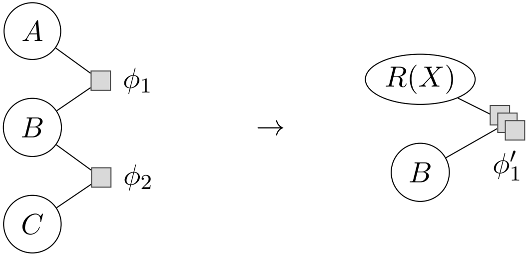
&= \frac{1}{Z} \cdot \sum_{a \in \mathcal{R}(A)} \phi_1(A = a, B) \cdot \sum_{c \in \mathcal{R}(C)} \phi_2(C = c, B) \\
&= \frac{1}{Z} \cdot \Big(\sum_{a \in \mathcal{R}(A)} \phi_1(A = a, B) \Big)^2 = \frac{1}{Z} \cdot \Big(\sum_{c \in \mathcal{R}(C)} \phi_2(C = c, B) \Big)^2
\end{aligned}
$$

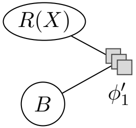
Parametric Factor Graphs

- Assumption: Symmetries in a graph
- Introduce logical variables to represent groups of random variables
  - $\mathcal{D}(X) = \{A, C\}$



David Poole (2003). »First-Order Probabilistic Inference«. *Proceedings of the Eighteenth International Joint Conference on Artificial Intelligence (IJCAI-2003)*. Morgan Kaufmann Publishers Inc., pp. 985–991.

Symmetries in Factor Graphs (Continued)

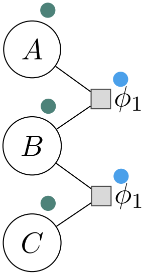


$R(X)$	B	$\phi'_1(R(X), B)$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

$$
\begin{aligned}
P(B) &= \frac{1}{Z} \cdot \Big(\sum_{a \in \mathcal{R}(A)} \phi_1(A = a, B) \Big)^2 \\
&= \frac{1}{Z} \cdot \Big(\sum_{c \in \mathcal{R}(C)} \phi_2(C = c, B) \Big)^2 \\
&= \frac{1}{Z} \cdot \Big(\sum_{r \in \mathcal{R}(R(X))} \phi'_1(R(X) = r, B) \Big)^{|\mathcal{D}(X)|}
\end{aligned}
$$

The Colour Passing Algorithm

- Assign colours to random variables depending on their ranges and evidence
- Assign colour to factors depending on their potentials
- Pass colours around to detect symmetries in the graph



Babak Ahmadi et al. (2013). »Exploiting Symmetries for Scaling Loopy Belief Propagation and Relational Training«. *Machine Learning* 92, pp. 91–132.

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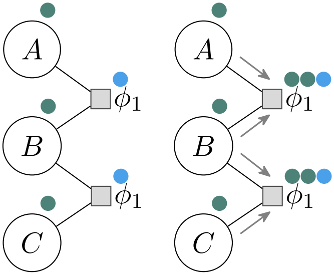

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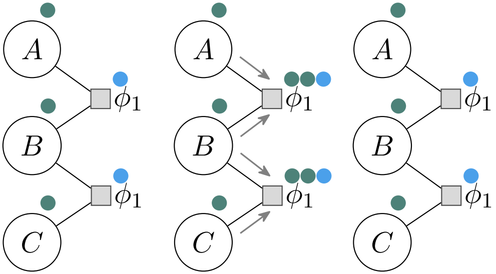

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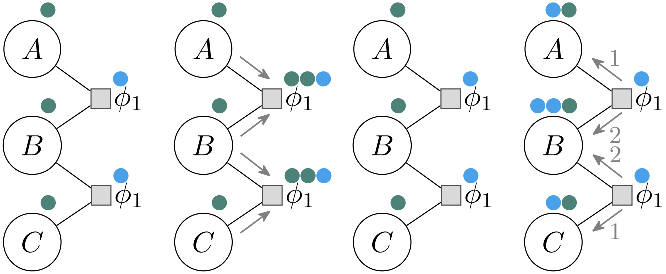

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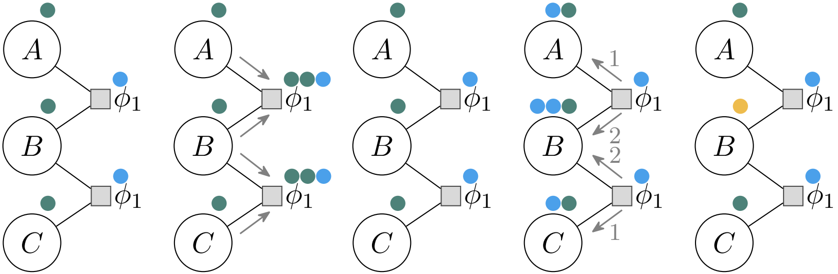

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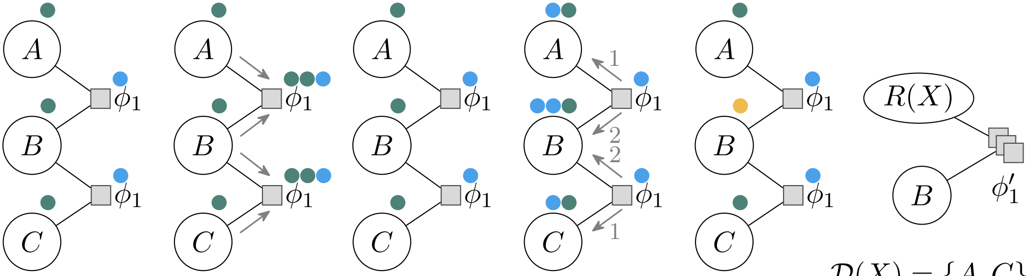


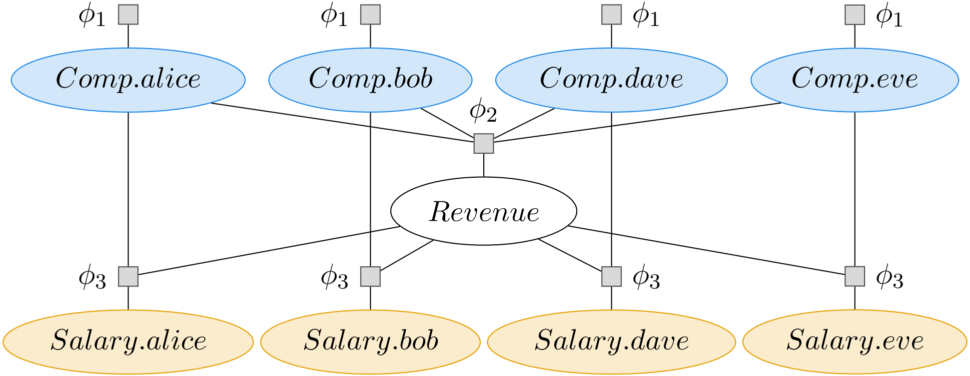
$$\mathcal{D}(X) = \{A, C\}$$

Babak Ahmadi et al. (2013). »Exploiting Symmetries for Scaling Loopy Belief Propagation and Relational Training«. *Machine Learning* 92, pp. 91–132.

Limitations of the Colour Passing Algorithm

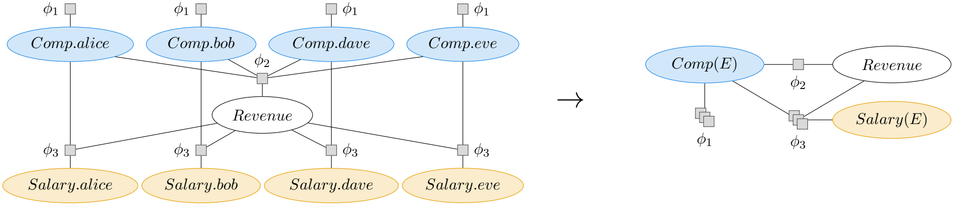
1. Commutative factors are not recognised
2. Argument orders must be fixed to detect identical factors
3. No logical variables are introduced

Handling Factor Graphs with Commutative Factors



Problem Setup

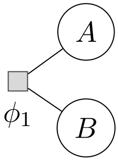
- Input: A factor graph G
- Output: A parametric factor graph entailing equivalent semantics as G
  - Under consideration of commutative factors
  - Independent of the order of factor's arguments



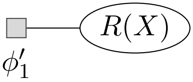
Commutative Factors

- Use histograms to detect commutativity of factors
- Each histogram must be mapped to a unique value
- If a factor is commutative, its neighbours might be grouped

A	B	$\phi_1(A, B)$
true	true	φ_1
true	false	φ_2
false	true	φ_2
false	false	φ_3

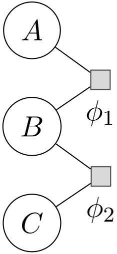


$\#_X[R(X)]$	$\phi'_1(\#_X[R(X)])$
$[2, 0]$	φ_1
$[1, 1]$	φ_2
$[0, 2]$	φ_3



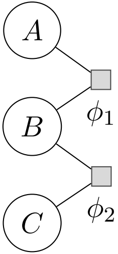
Permuted Factors

- Both factor graphs entail equivalent semantics
- Histograms can be used as a filter condition to find permutations



A	B	$\phi_1(A, B)$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

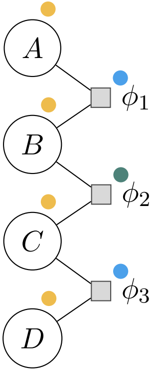
C	B	$\phi_2(C, B)$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4



A	B	$\phi_1(A, B)$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

B	C	$\phi_2(B, C)$
true	true	φ_1
true	false	φ_3
false	true	φ_2
false	false	φ_4

The Advanced Colour Passing Algorithm



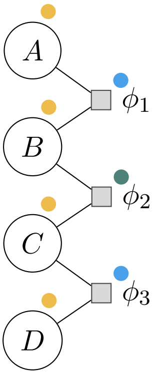
A	B	$\phi_1(A, B)$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

C	D	$\phi_3(C, D)$
true	true	φ_1
true	false	φ_3
false	true	φ_2
false	false	φ_4

B	C	$\phi_2(B, C)$
true	true	φ_5
true	false	φ_6
false	true	φ_6
false	false	φ_7

Malte Luttermann, Tanya Braun, et al. (2024). »Colour Passing Revisited: Lifted Model Construction with Commutative Factors«. *Proceedings of the Thirty-Eighth AAAI Conference on Artificial Intelligence (AAAI-2024)*. AAAI Press, pp. 20500–20507.

The Advanced Colour Passing Algorithm



A	B	$\phi_1(A, B)$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

D	C	$\phi_3(D, C)$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

B	C	$\phi_2(B, C)$
true	true	φ_5
true	false	φ_6
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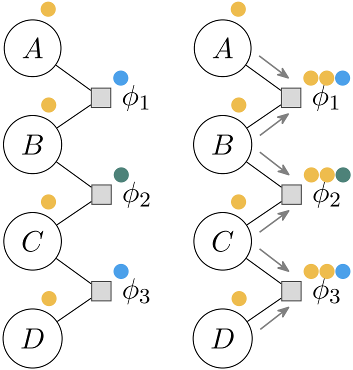

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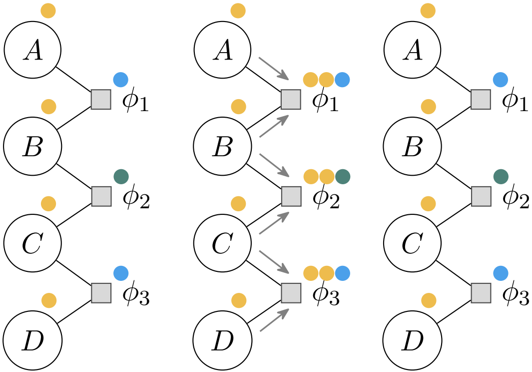

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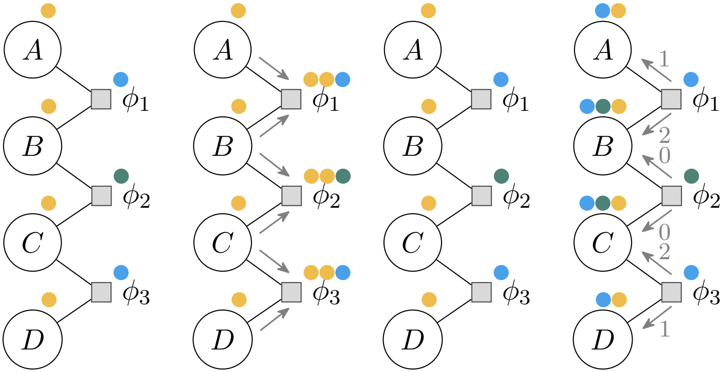

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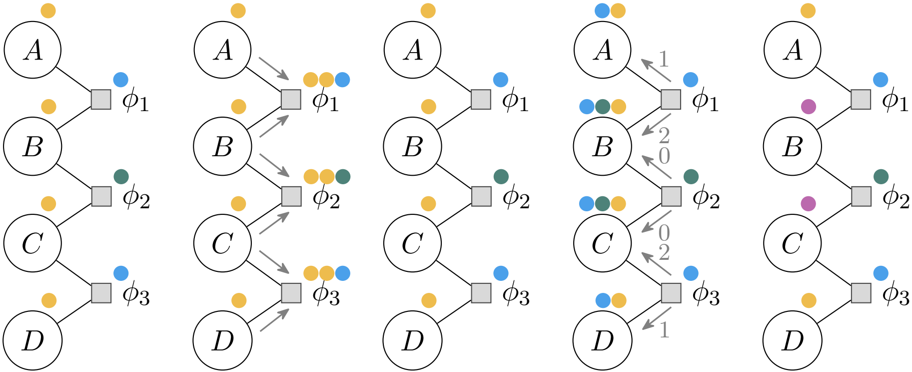

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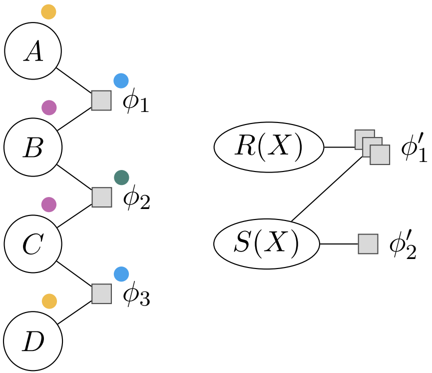


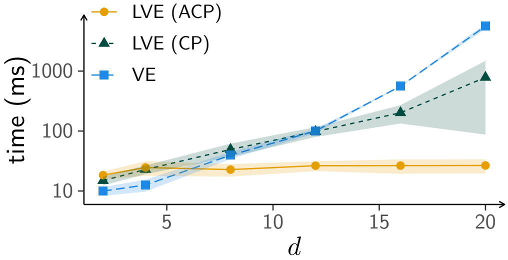
$R(X)$	$S(X)$	$\phi'_1(R(X), S(X))$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

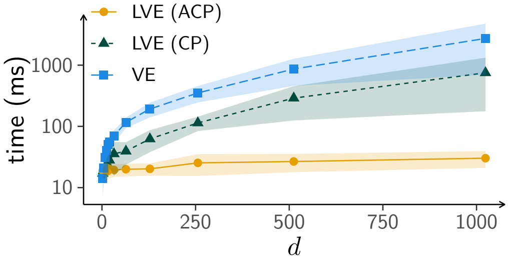
$\#_X[S(X)]$	$\phi'_2(\#_X[S(X)])$
$[2, 0]$	φ_5
$[1, 1]$	φ_6
$[0, 2]$	φ_7

Malte Luttermann, Tanya Braun, et al. (2024). »Colour Passing Revisited: Lifted Model Construction with Commutative Factors«. *Proceedings of the Thirty-Eighth AAAI Conference on Artificial Intelligence (AAAI-2024)*. AAAI Press, pp. 20500–20507.

Empirical Evaluation of Advanced Colour Passing

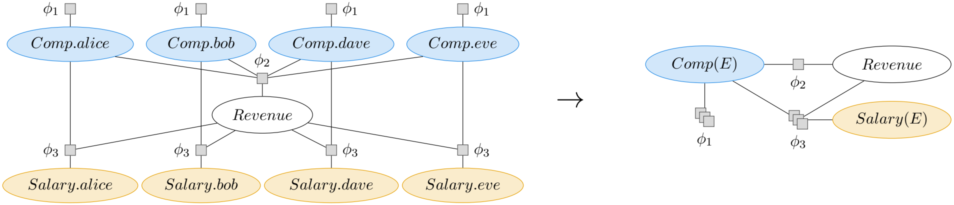
- Comparison of run times for lifted inference
- Left: Factor graphs with 1 commutative factor (no permuted argument orders)
- Right: Factor graphs where the arguments of 3 percent of the factors are permuted (no commutative factors)



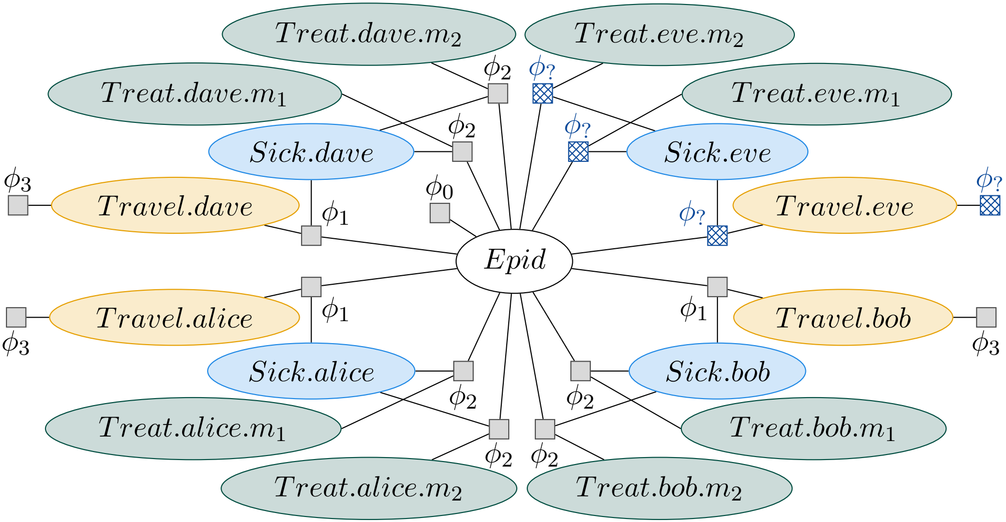


Intermediate Summary

- Handling commutative factors
- Detect identical factors independent of their argument orders
- Algorithm to introduce logical variables

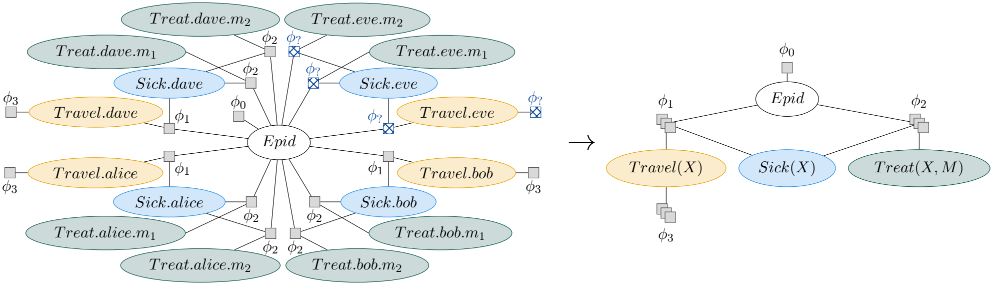


Handling Factor Graphs with Unknown Factors



Problem Setup

- Input: A factor graph G possibly containing unknown factors
- Output: A lifted representation of G



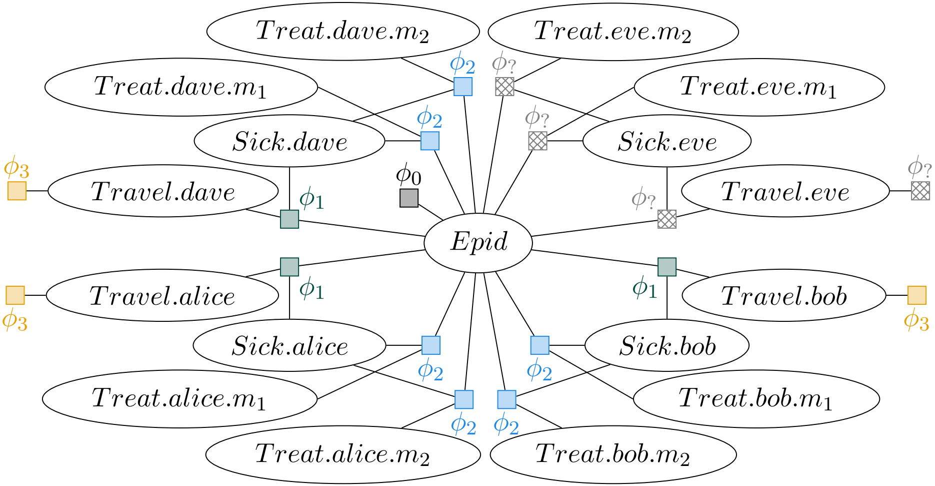
Lifting Factor Graphs with Some Unknown Factors (LIFAGU) Algorithm

- What colour to assign to the unknown factors?
  - Potentials are missing
  - Only available information: Surrounding graph structure
- General idea:
  1. Known factors are coloured according to their potentials
  2. Unknown factors are coloured according to their 2-step neighbourhood
  3. Assign unknown factors and known factors the same colour if their 2-step neighbourhoods are symmetric
  4. Run the standard colour passing algorithm

Malte Luttermann, Ralf Möller, and Marcel Gehrke (2023). »Lifting Factor Graphs with Some Unknown Factors«. *Proceedings of the Seventeenth European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU-2023)*. Springer, pp. 337–347.

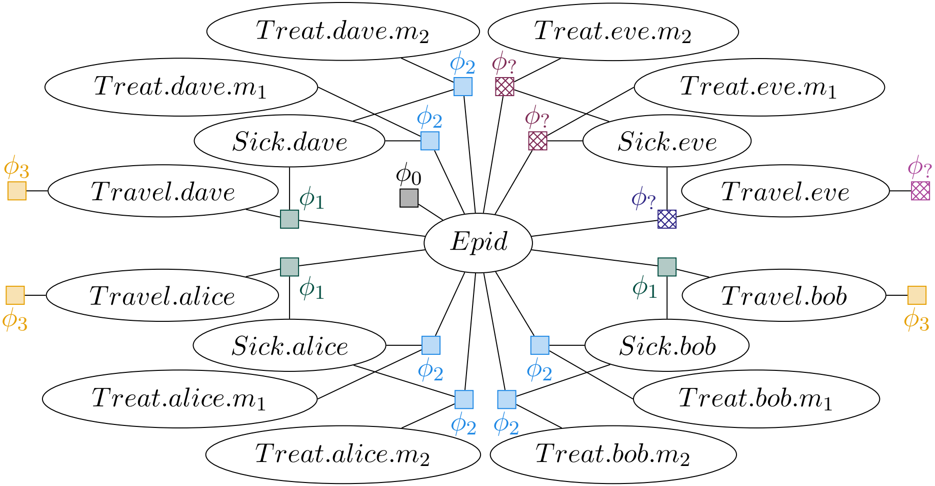
Lifting Factor Graphs with Some Unknown Factors (LIFAGU) Algorithm

1. Known factors are coloured according to their potentials



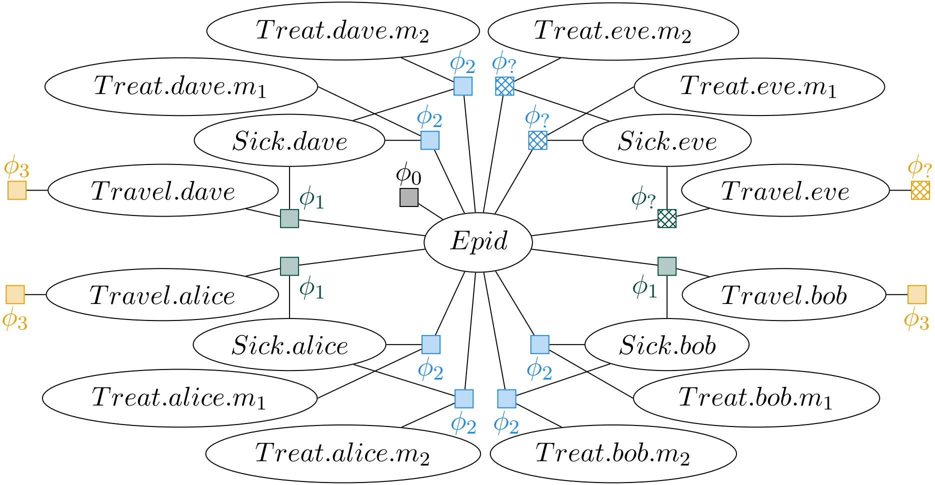
Lifting Factor Graphs with Some Unknown Factors (LIFAGU) Algorithm

2. Unknown factors are coloured according to their 2-step neighbourhood



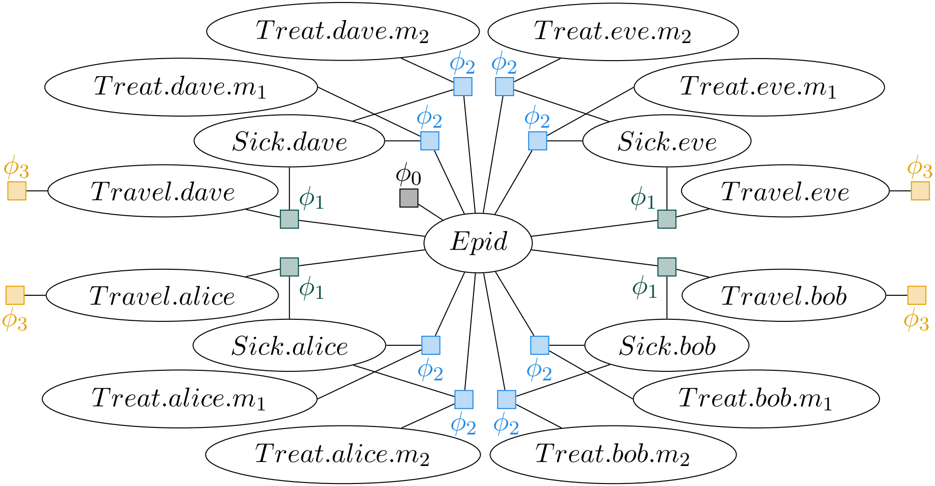
Lifting Factor Graphs with Some Unknown Factors (LIFAGU) Algorithm

3. Assign unknown factors and known factors the same colour if their 2-step neighbourhoods are symmetric



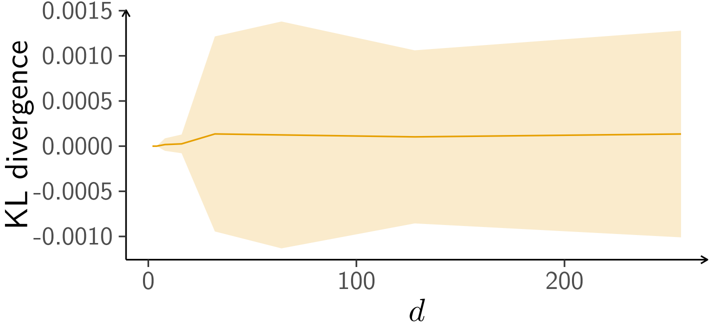
Lifting Factor Graphs with Some Unknown Factors (LIFAGU) Algorithm

4. Run the standard colour passing algorithm



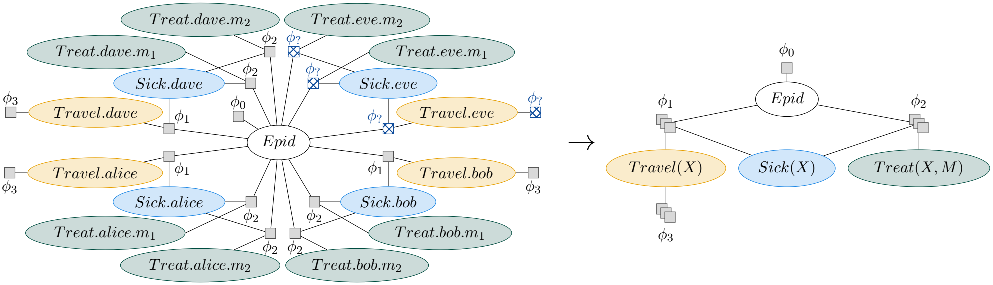
Empirical Evaluation of LIFAGU

- Generate factor graphs where all factors are known
- Between 3 and 5 (randomly chosen) cohorts of random variables
- Randomly remove potentials of 5 to 10 percent of the factors
- Run LIFAGU to obtain a lifted representation
- Perform probabilistic inference on the ground truth and the lifted representation



Intermediate Summary

- Construct a lifted representation for factor graphs with unknown factors
- Transfer known potentials to unknown factors
- Ensure a well-defined semantics and allow for lifted inference



Conclusion and Outlook

What has been done:

- Learn a parametric factor graph from a propositional factor graph
    - Handle commutative factors
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- Limitation: The propositional factor graph must be constructed

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What has to be done in future work:

- Learn a parametric factor graph directly from a relational database
- Allow for causal inference in parametric factor graphs
- Sample from parametric factor graphs to generate synthetic data

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