Efficient Detection of Commutative Factors in Factor Graphs

PGM 2024 – Nijmegen, Netherlands

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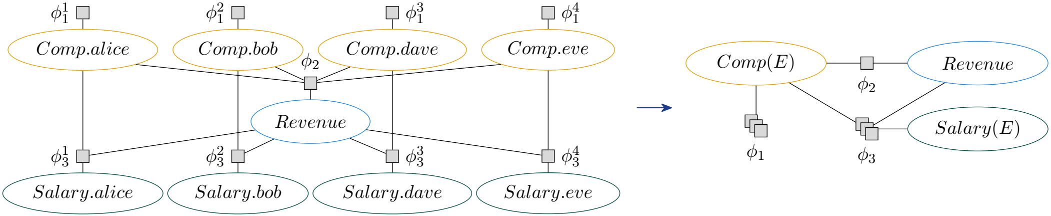
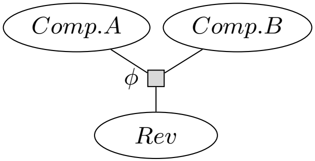
Motivation

- Lifting exploits symmetries in factor graphs to speed up probabilistic inference
- Lifting uses a representative of indistinguishable individuals for computations



Problem Setup I

- Goal: Efficiently detect commutative factors
- Commutative factors have symmetries within themselves due to their arguments being exchangeable



$Comp.A$	$Comp.B$	Rev	$\phi(Comp.A, Comp.B, Rev)$
true	true	true	φ_1
true	true	false	φ_4
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false	true	true	φ_2
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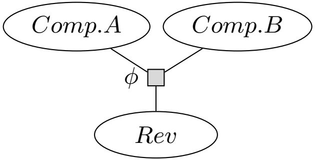

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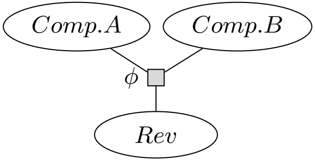

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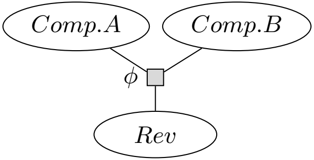

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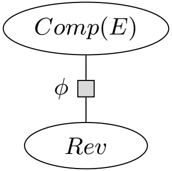

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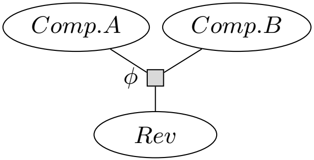

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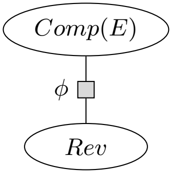

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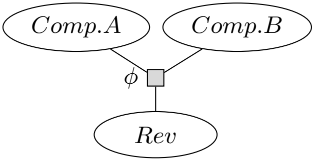

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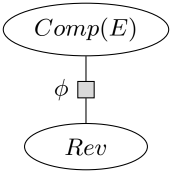

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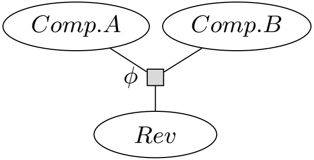

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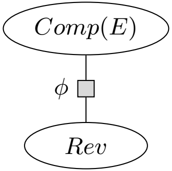

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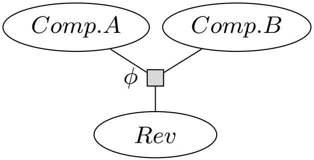

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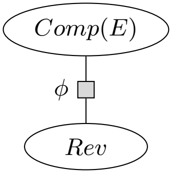

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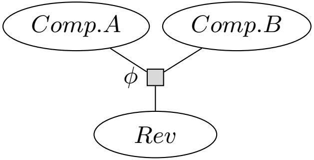

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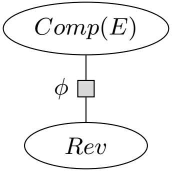

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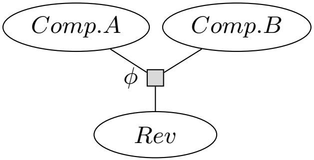

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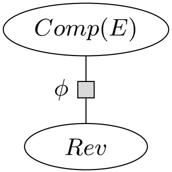

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Previous Work and Our Contributions

Previous work (Luttermann et al., 2024):

- Advanced Colour Passing algorithm to construct a lifted representation
- Iterate over all subsets of arguments to find commutative arguments
- Number of iterations for a factor with n arguments is in $O(2^n)$

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Our contributions:

- Theoretical guarantees: *Buckets* to avoid iterating over all subsets
- Practical algorithm: Detection of Commutative Factors (DECOR) algorithm

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Buckets

- Buckets count the occurrences of specific range values in an assignment
- Each potential belongs to *exactly one* bucket
- Each bucket contains *at least one* potential

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b	$\phi(b)$
$[3,0]$	$\langle\varphi_1\rangle$
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Properties of Buckets I

For any subset $\boldsymbol{S}$ of commutative arguments:

- $|\boldsymbol{S}| \leq \min_{b \in \{b \mid b \in \mathcal{B}(\phi) \wedge |\phi(b)|>1\}} \max_{\varphi \in \phi(b)} \operatorname{count}(\phi(b), \varphi)$, i.e.,
- in each bucket b with $|\phi(b)| > 1$, there is a potential occurring at least $|\boldsymbol{S}|$ times

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Properties of Buckets II

For a group of identical potentials in a bucket:

- Intersection of their corresponding assignments yields candidates
- E.g., for φ_2's in $[2, 1]$: $(\text{true}, \text{false}, \text{true}) \cap (\text{false}, \text{true}, \text{true}) = (\emptyset, \emptyset, \text{true})$

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The DECOR Algorithm

- Iterate over buckets and compute candidates for commutative arguments

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Initial candidates: $\{\{A,B,R\}\}$

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$[3, 0]$ $(|\phi(b)| < 2)$: $\{\{A, B, R\}\}$

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$$
\begin{aligned}
& (\mathrm{true}, \mathrm{false}, \mathrm{true}) \\
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=\ & (\emptyset, \quad \emptyset, \quad \mathrm{true})
\end{aligned}
$$

Initial candidates: $\{\{A,B,R\}\}$

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Initial candidates: $\{\{A, B, R\}\}$

$[3, 0]$ $(|\phi(b)| < 2)$: $\{\{A, B, R\}\}$

$[2, 1]$: $\{\{A, B\}\}$

$[1, 2]$: $\{\{A, B\}\}$

The DECOR Algorithm

- Iterate over buckets and compute candidates for commutative arguments

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Initial candidates:	$\{\{A, B, R\}\}$
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$[1, 2]$:	$\{\{A, B\}\}$
$[0, 3]$ $(\lvert\phi(b)\rvert < 2)$:	$\{\{A, B\}\}$

Theoretical Guarantees of the DECOR Algorithm

- Avoids »naive« iteration over all 2^n subsets of arguments
- Time complexity is upper-bounded depending on the number of groups of identical potentials in the buckets

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Theoretical Guarantees of the DECOR Algorithm

- Avoids »naive« iteration over all 2^n subsets of arguments
- Time complexity is upper-bounded depending on the number of groups of identical potentials in the buckets
- In practice, there are two possible scenarios:

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$[3,0]$	$\langle \varphi_1 \rangle$
$[2,1]$	$\langle \varphi_4, \varphi_2, \varphi_2 \rangle$
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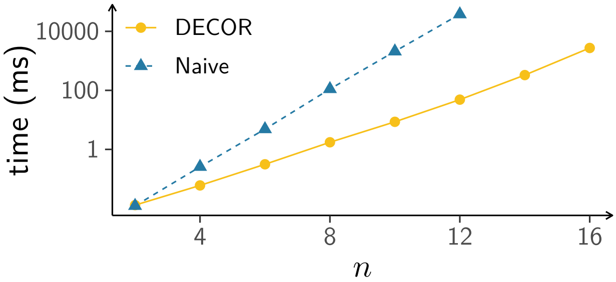
1. A factor contains commutative arguments
   - Potential values are likely to contain a single group of duplicates
2. A factor contains no commutative arguments
   - Potential values are likely to be distinct

Experiments

- Comparison of run times of DECOR and the »naive« approach
- Average over factors with $k \in \{0, 2, \lfloor \frac{n}{2} \rfloor, n-1, n\}$ commutative arguments
- Timeout after five minutes per instance



Summary

- Problem of efficiently detecting commutative factors solved
- Upper bound on the number of iterations depending on the number of groups of identical potential values within the buckets of a factor
- The DECOR algorithm effectively exploits this upper bound in practice