UNIVERSITY OF LÜBECK
INSTITUTE OF INFORMATION SYSTEMS

Colour Passing Revisited: Lifted Model Construction with Commutative Factors

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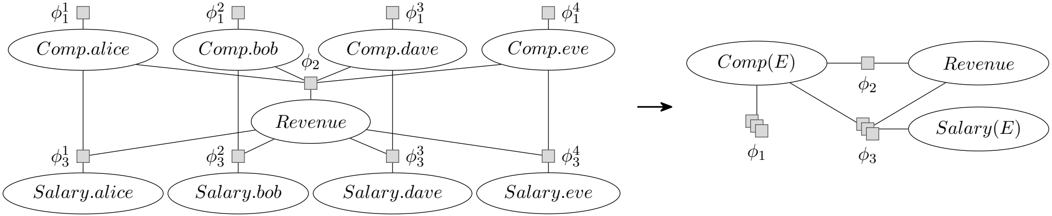
Problem Setup

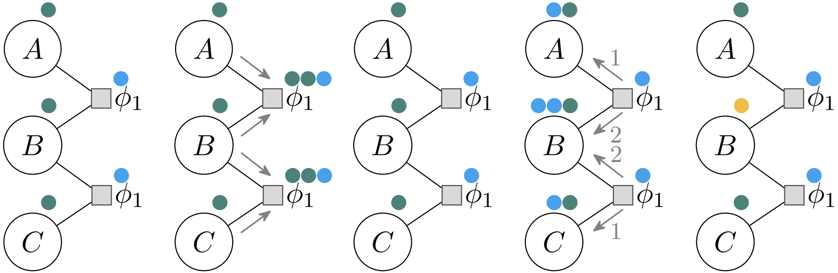
- Input: A factor graph G
- Output: A parametric factor graph entailing equivalent semantics as G
  - Under consideration of commutative factors
  - Independent of the order of factor's arguments



Previous Work: Colour Passing Algorithm

(Kersting, Ahmadi, and Natarajan, 2009; Ahmadi et al., 2013)

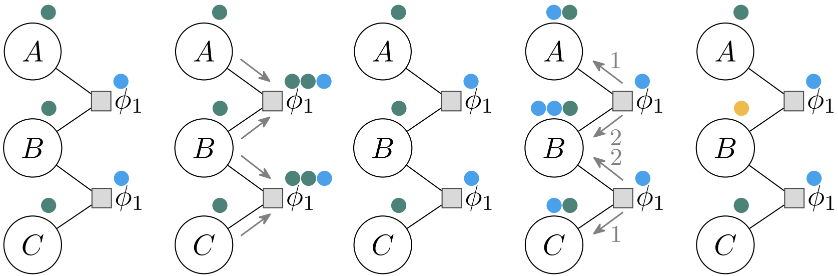
- Assign colours to random variables depending on their ranges and evidence
- Assign colour to factors depending on their potentials
- Pass colours around to detect symmetries in the graph



Limitations of the Colour Passing Algorithm

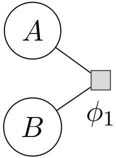
(Kersting, Ahmadi, and Natarajan, 2009; Ahmadi et al., 2013)

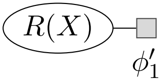
1. Commutative factors are not recognised
2. Relies on fixed argument orders to detect factor equivalence
3. No logical variables are introduced



Commutative Factors

- Detecting commutative factors allows to further compress the graph
- Histograms to efficiently detect and represent commutative factors
- Also applicable to a subset of arguments





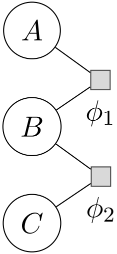
A	B	$\phi_1(A, B)$
true	true	φ_1
true	false	φ_2
false	true	φ_2
false	false	φ_3

$\longrightarrow$

$\#_X[R(X)]$	$\phi'_1(\#_X[R(X)])$
$[2, 0]$	φ_1
$[1, 1]$	φ_2
$[0, 2]$	φ_3

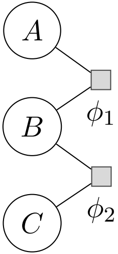
Permuted Factors

- Histograms are an efficient filter condition to check for factor equivalence
- Histograms determine possible permutations of factor arguments



A	B	$\phi_1(A, B)$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

C	B	$\phi_2(C, B)$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

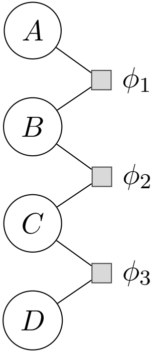


A	B	$\phi_1(A, B)$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

B	C	$\phi_2(B, C)$
true	true	φ_1
true	false	φ_3
false	true	φ_2
false	false	φ_4

The Advanced Colour Passing Algorithm

Checking for Factor Equivalence



A	B	$\phi_1(A, B)$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

C	D	$\phi_3(C, D)$
true	true	φ_1
true	false	φ_3
false	true	φ_2
false	false	φ_4

B	C	$\phi_2(B, C)$
true	true	φ_5
true	false	φ_6
false	true	φ_6
false	false	φ_7

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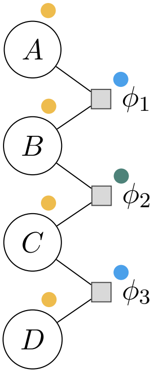
Initial Colour Assignments



A	B	$\phi_1(A,B)$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

D	C	$\phi_3(D,C)$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

B	C	$\phi_2(B,C)$
true	true	φ_5
true	false	φ_6
false	true	φ_6
false	false	φ_7

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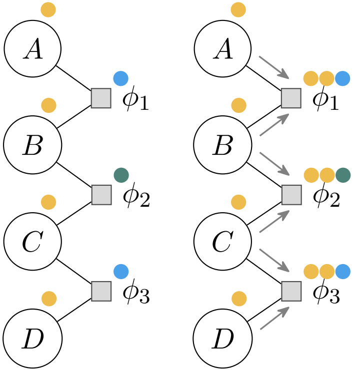
Colour Passing from Variables to Factors



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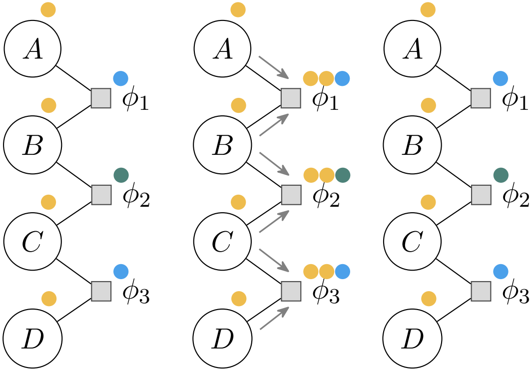
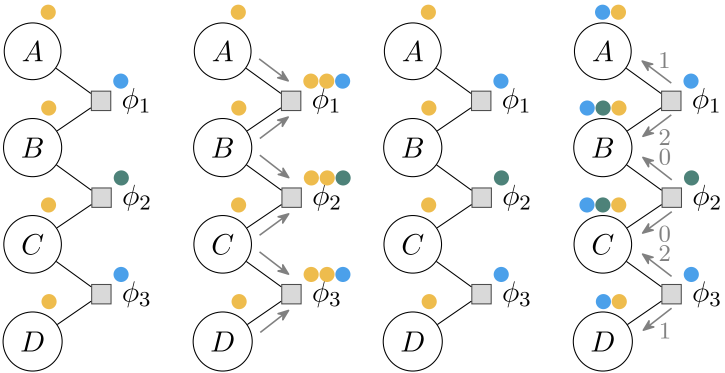
Recolouring of Factors



The Advanced Colour Passing Algorithm

Colour Passing from Factors to Variables



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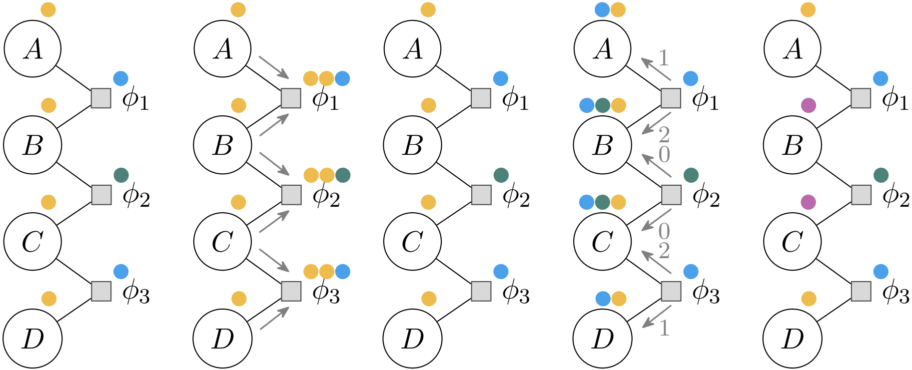
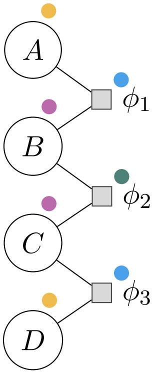
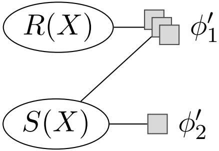
Recolouring of Variables



The Advanced Colour Passing Algorithm

Introduction of Logical Variables



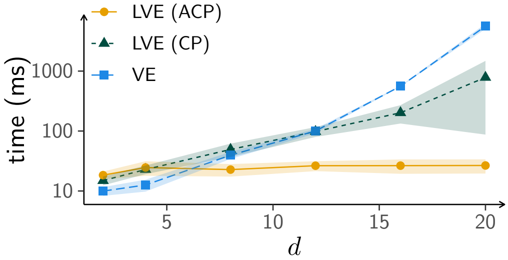


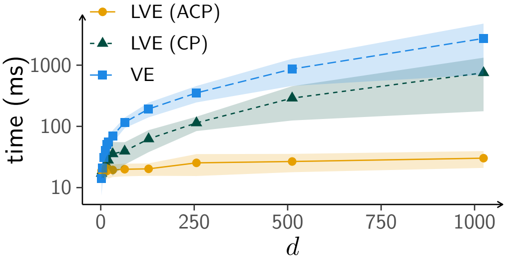
$R(X)$	$S(X)$	$\phi'_1(R(X), S(X))$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

$\#_X[S(X)]$	$\phi'_2(\#_X[S(X)])$
$[2, 0]$	φ_5
$[1, 1]$	φ_6
$[0, 2]$	φ_7

Empirical Evaluation of Advanced Colour Passing

- Comparison of run times for lifted inference
- Left: Factor graphs with 1 commutative factor (no permuted argument orders)
- Right: Factor graphs where the arguments of 3 percent of the factors are permuted (no commutative factors)

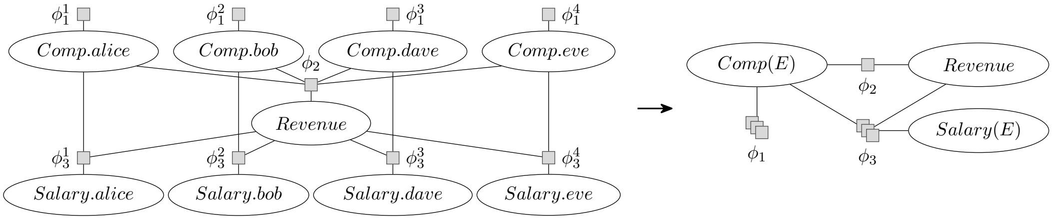




Summary

Limitations of Colour Passing solved:

1. Histograms to efficiently detect and compactly represent commutative factors
2. Histograms to efficiently identify factor equivalence
3. Logical variables for the class of domain-liftable models



References

- Babak Ahmadi, Kristian Kersting, Martin Mladenov, and Sriraam Natarajan (2013). »Exploiting Symmetries for Scaling Loopy Belief Propagation and Relational Training«. *Machine Learning* 92, pp. 91–132.
- Kristian Kersting, Babak Ahmadi, and Sriraam Natarajan (2009). »Counting Belief Propagation«. *Proceedings of the Twenty-Fifth Conference on Uncertainty in Artificial Intelligence (UAI-09)*. AUAI Press, pp. 277–284.