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Capturing Inter-Individual Differences in Cognitive Processes via Hierarchical Clustering in Dynamic Bayesian Networks

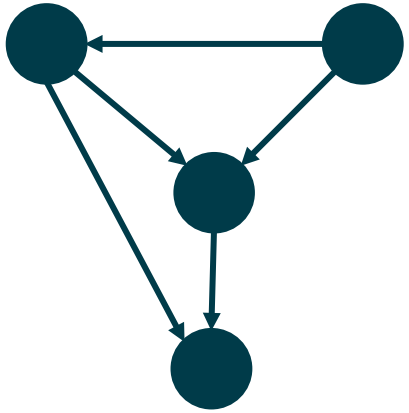
Rebecca von Engelhardt and Malte Luttermann

09.10.2026

ACS 2026

Decision Making in Social Interactions





Different Modelling Approaches

Probabilistic graphical models
(e.g., DBN)

Cognitive architectures
(e.g., ACT-R)

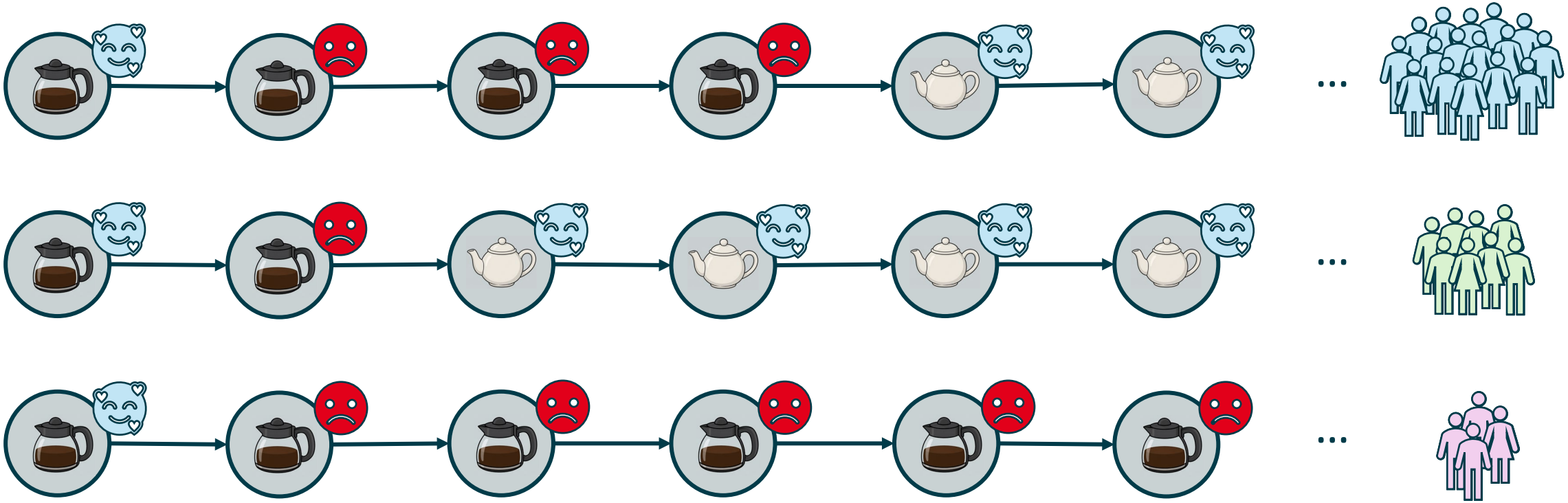
Qualitative models

Traditional learning models
(e.g., RW)

…

Finding Groups With Similar Patterns



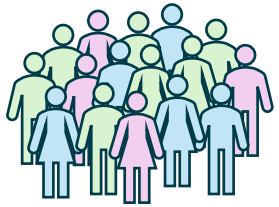


How?!









Algorithm 1 Hierarchical Ordering of ε-Equivalent Groups (Speller et al., 2025)

Input: A set $\Phi = \{\phi^1, \ldots, \phi^m\} \subset \mathbb{R}^{n \times m}_{\geq 0}$ of real-valued vectors of dimension n each.
Output: An ordered nested list $\mathcal{L}$ of lists and an ordered vector $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_{m-1})$ with $\varepsilon_i \leq \varepsilon_{i+1}$ for all i.

▷ Phase I: Generate upper triangular matrix Λ
Initialise $\Lambda \in \mathbb{R}^{m \times m}$ with zeros
for $i = 1$ to $m - 1$ **do**
  for $j = i + 1$ to m **do**
    Compute $D_{\text{RMSE}}(\phi^i, \phi^j) =: \varepsilon_{i,j}$
    Store result in Λ_{ij}
Save Λ
▷ Phase II: Generate ordered list $\mathcal{L}$ and vector ε
Initialise empty list $\mathcal{L} \leftarrow []$
Initialise vector $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_{m-1})$ with zeros
Initialise active index set $\mathcal{A} \leftarrow \{1, \ldots, m\}$
Initialise active matrix $\tilde{\Lambda} := \Lambda$
for $\ell = 1$ to $m - 1$ **do**
  Find $(i', j') = \arg\min\{\tilde{\Lambda}_{ij} \mid i < j,\ i, j \in \mathcal{A}\}$
  Save $\varepsilon_\ell := \tilde{\Lambda}_{i'j'}$
  if i' appears in direct parent group $G_p \in \mathcal{L}$ **then**
    if j' appears in direct parent group $G'_p \in \mathcal{L}$ **then**
      Update G_p and G'_p as one new group $[G_p, G'_p, \ell]$
    else
      Update G_p as $G_p = [G_p, j', \ell]$
  else if j' appears in direct parent group $G_p \in \mathcal{L}$ **then**
    Update G_p as $G_p = [G_p, i', \ell]$
  else ▷ Neither i' nor j' appears in any parent group
    Append $[[i', j', \ell]]$ to $\mathcal{L}$
  for $k \in \mathcal{A} \setminus \{i', j'\}$ **do**
    if $k > j'$ **then**
      $\tilde{\Lambda}_{i'k} \leftarrow \max\{\tilde{\Lambda}_{i'k}, \tilde{\Lambda}_{j'k}\}$
    else if $k < i'$ **then**
      $\tilde{\Lambda}_{ki'} \leftarrow \max\{\tilde{\Lambda}_{ki'}, \tilde{\Lambda}_{kj'}\}$
    else if $i' < k < j'$ **then**
      $\tilde{\Lambda}_{i'k} \leftarrow \max\{\tilde{\Lambda}_{i'k}, \tilde{\Lambda}_{kj'}\}$
  Remove j' from active set: $\mathcal{A} \leftarrow \mathcal{A} \setminus \{j'\}$



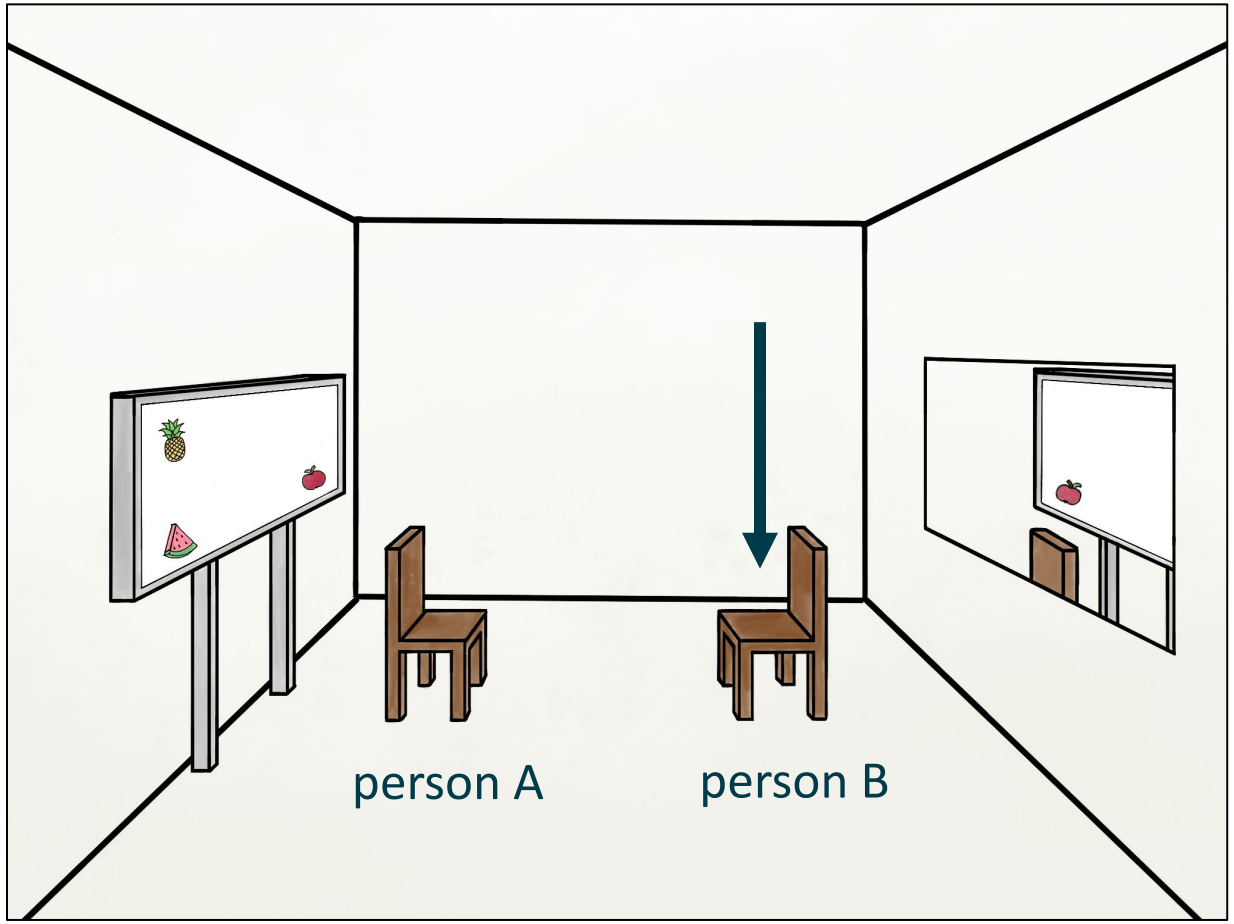








Experiment

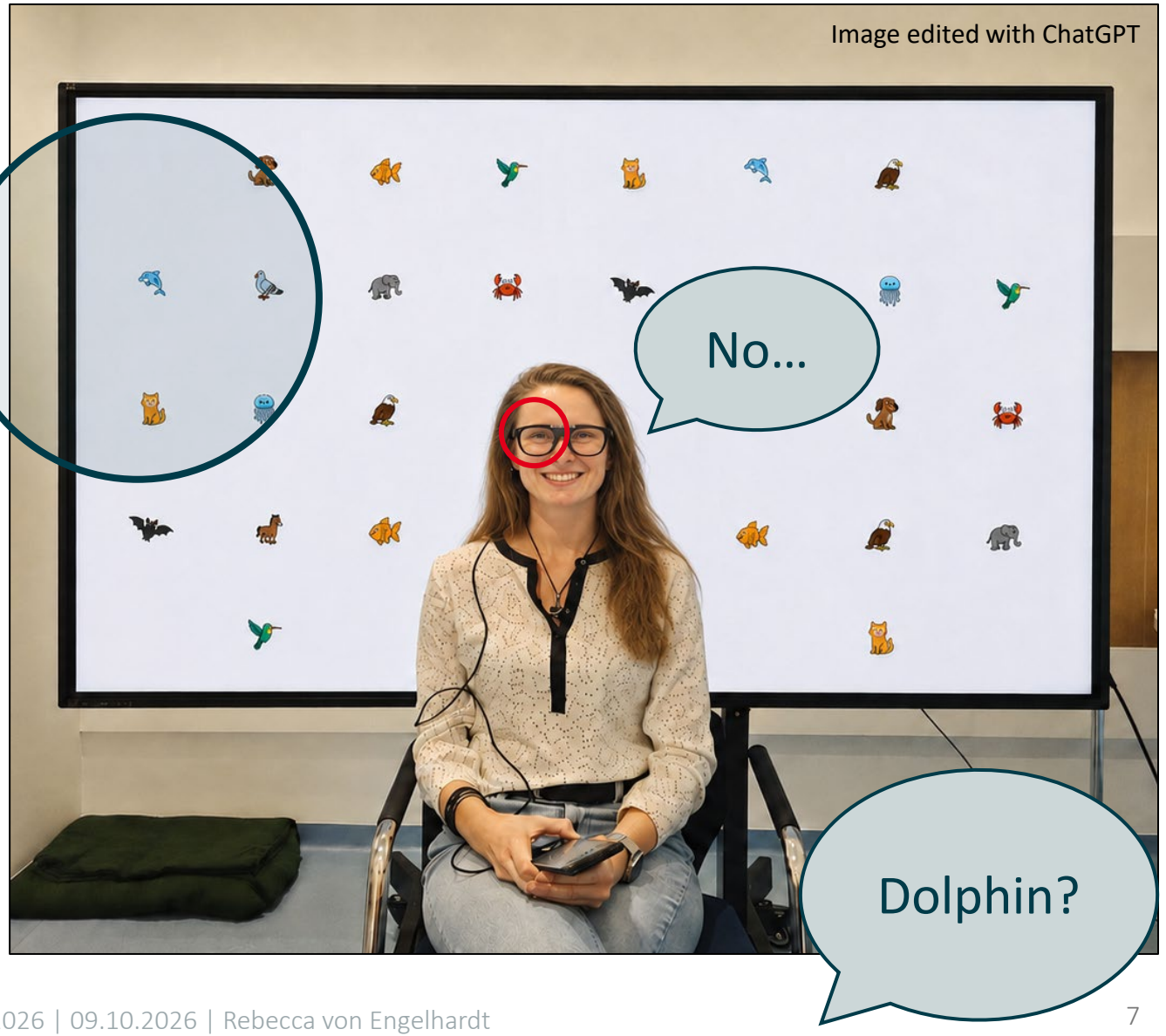






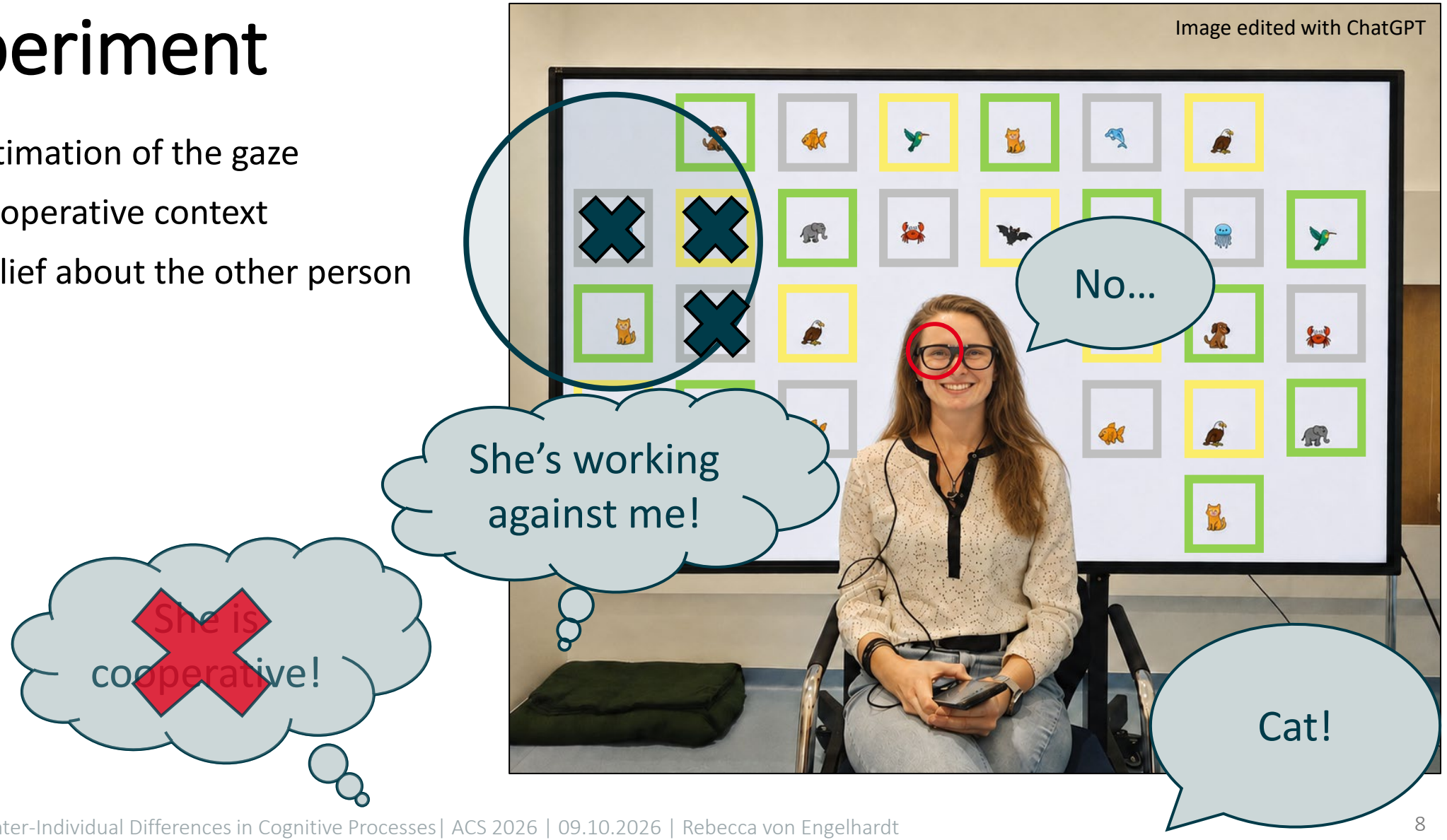
Experiment

1. Estimation of the gaze



Experiment

1. Estimation of the gaze
2. Cooperative context
3. Belief about the other person

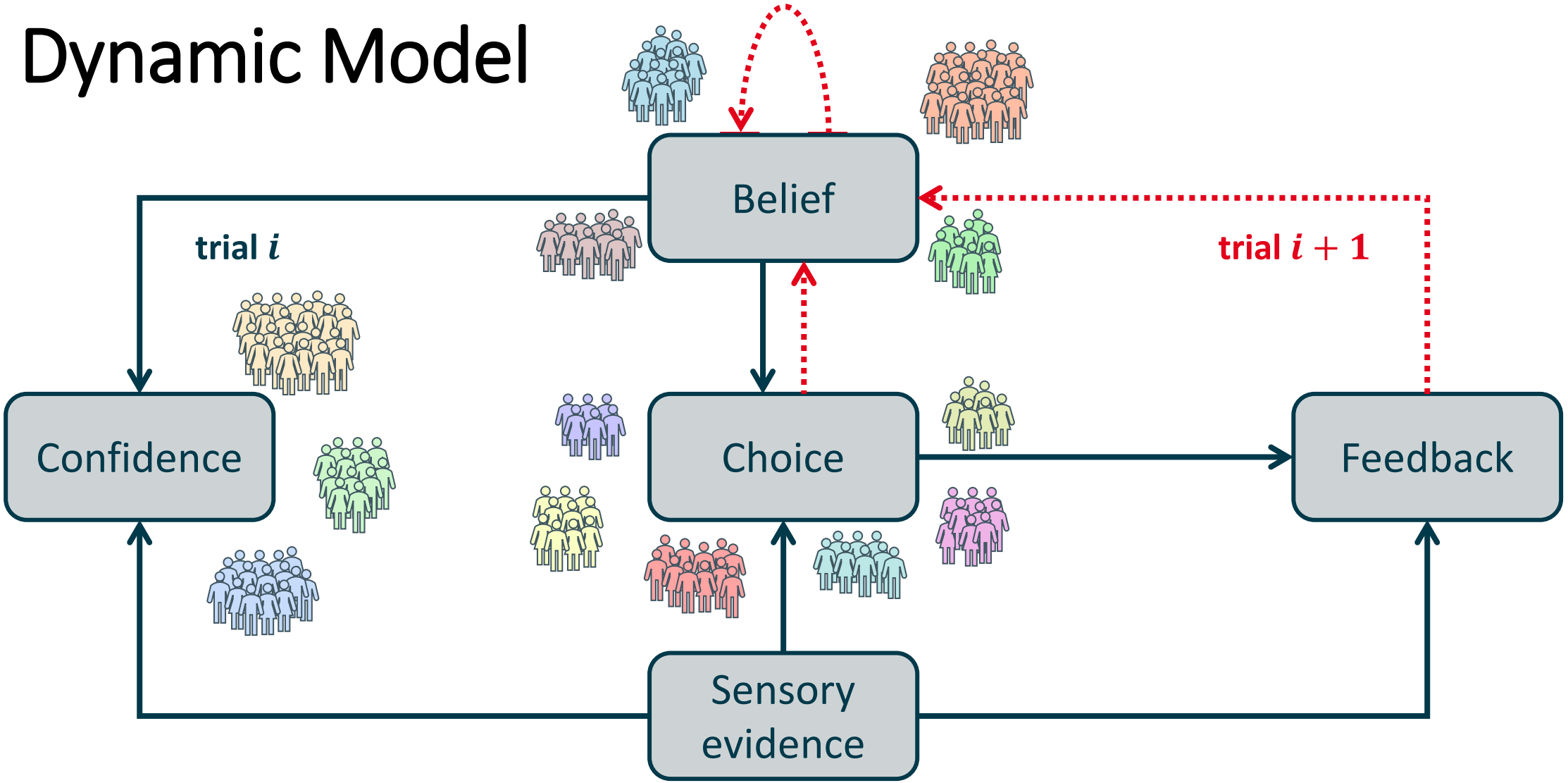


Experiment

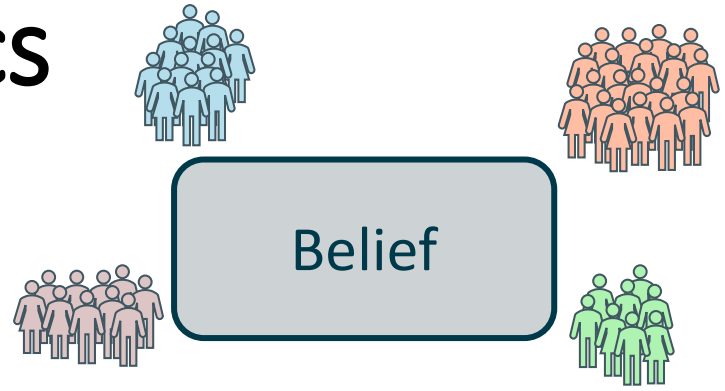
1. Estimation of the gaze
2. Cooperative context
3. Belief about the other person
4. Belief updating

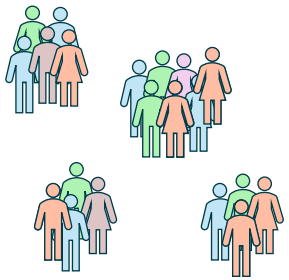


Dynamic Model



Distance Metrics





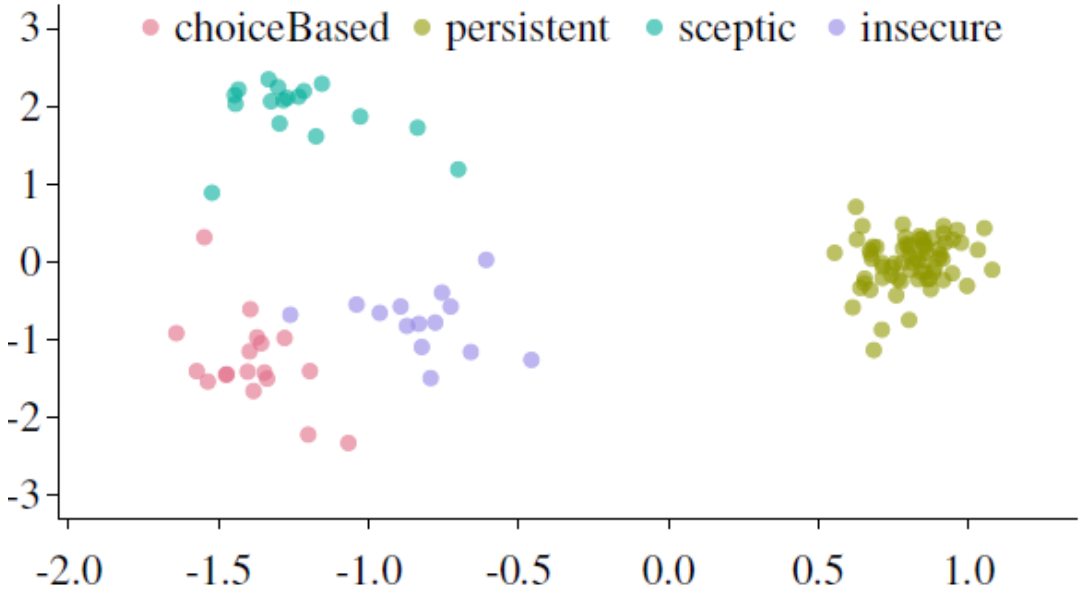
RMSE

$Choice_t$	$Belief_t$	$Feedback_t$	$P(Belief_{t+1} = high \mid Choice_t, Belief_t, Feedback_t)$	…	…
high	high	correct	1.00	0.00	0.00
high	high	incorrect	0.89	0.10	0.01
…	…	…	…	…	…

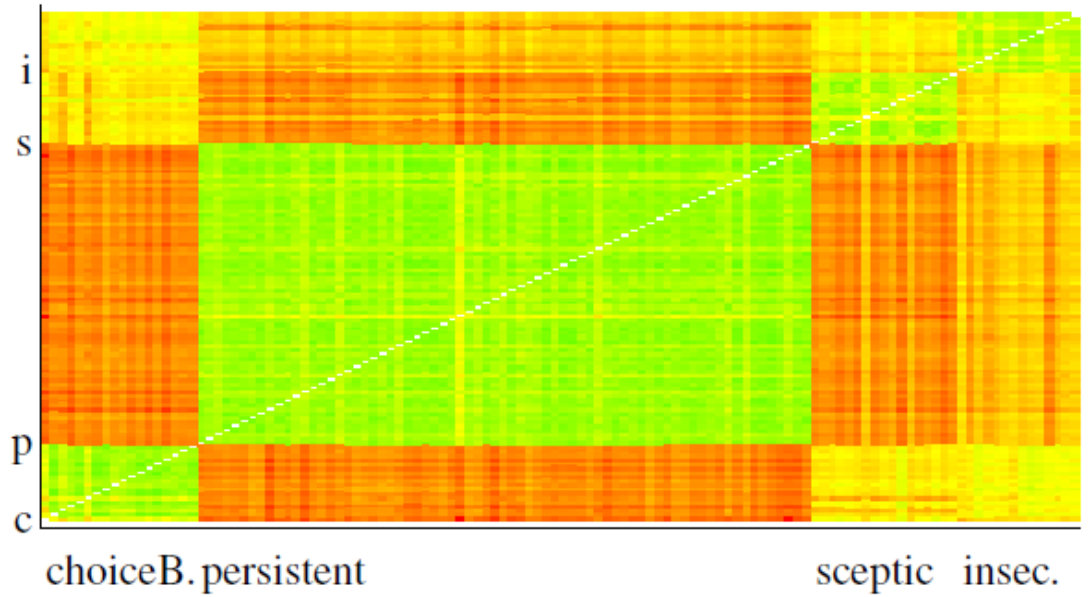
$Choice_t$	$Belief_t$	$Feedback_t$	$P(Belief_{t+1} = high \mid Choice_t, Belief_t, Feedback_t)$	…	…
high	high	correct	0.97	0.02	0.01
high	high	incorrect	0.91	0.09	0.00
…	…	…	…	…	…

Visualise Distances: Belief

RMSE distances in a 2D space



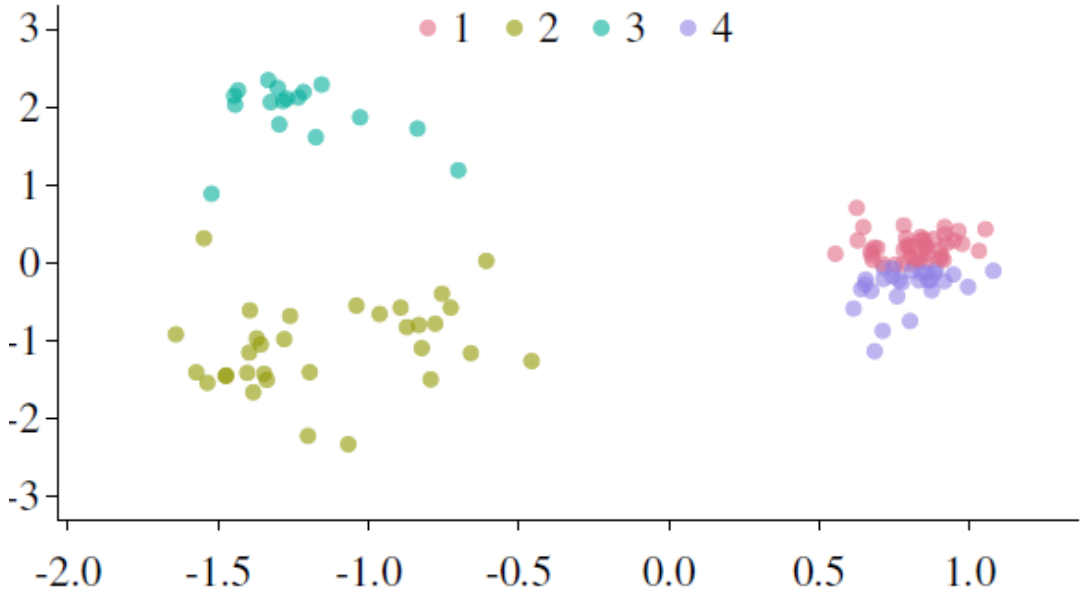
Heatmap



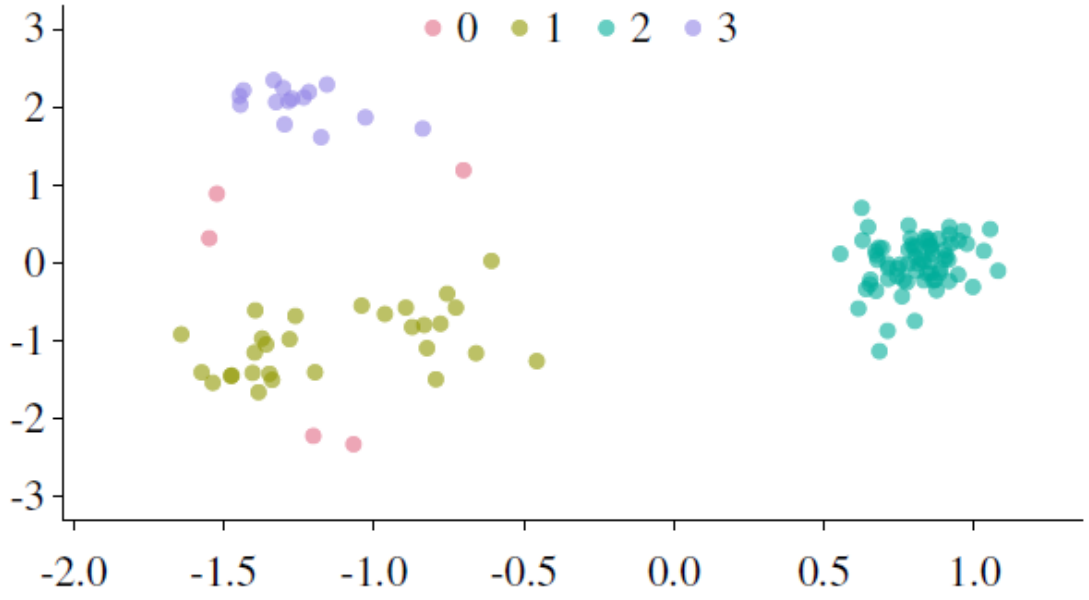
N = 120

K-Means and DBSCAN

K-Means

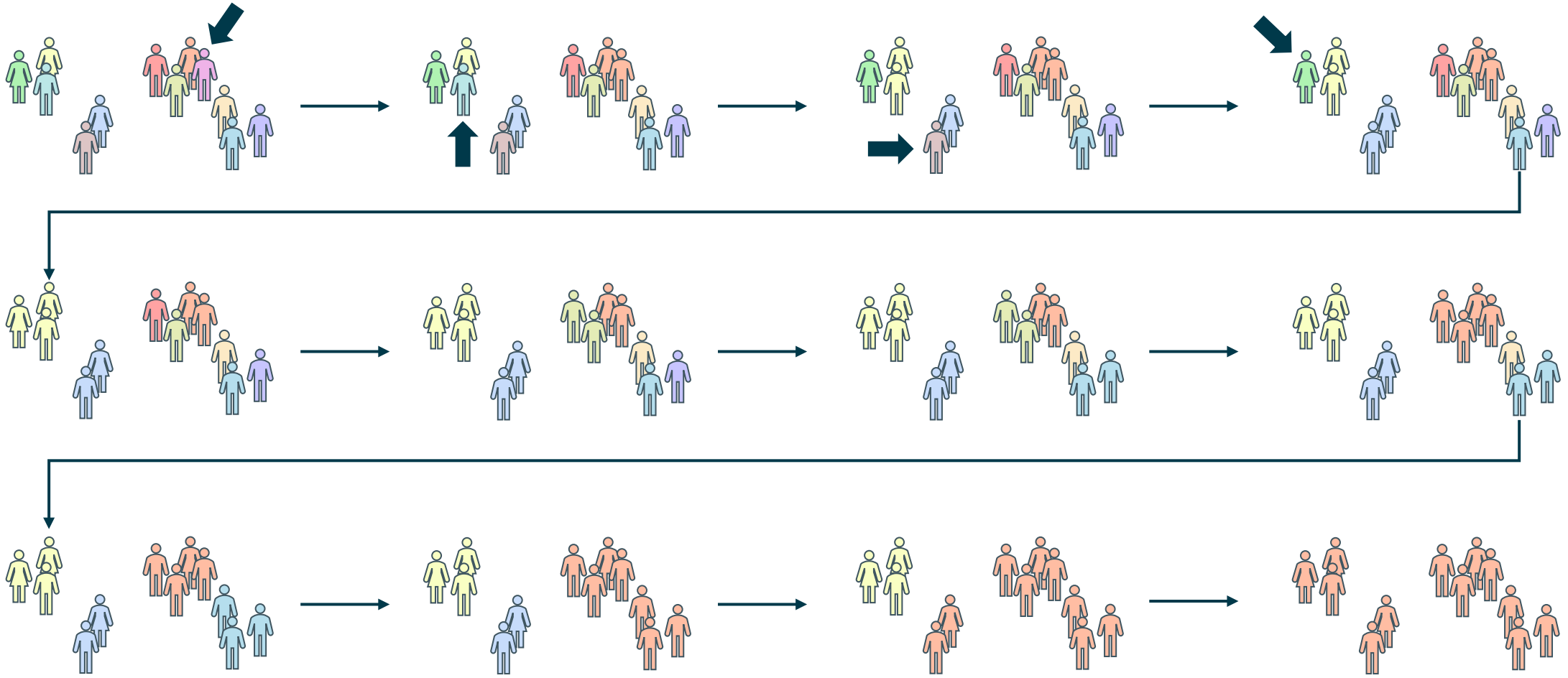


DBSCAN

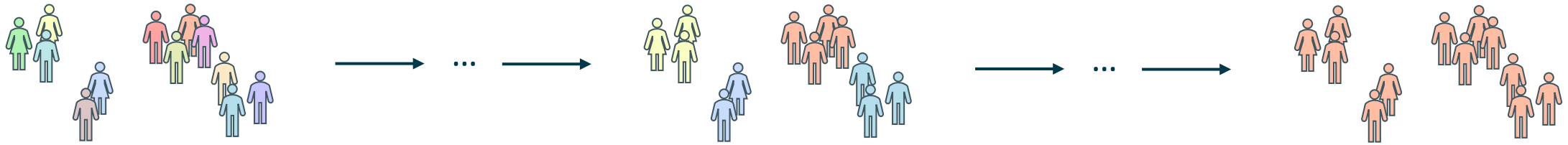


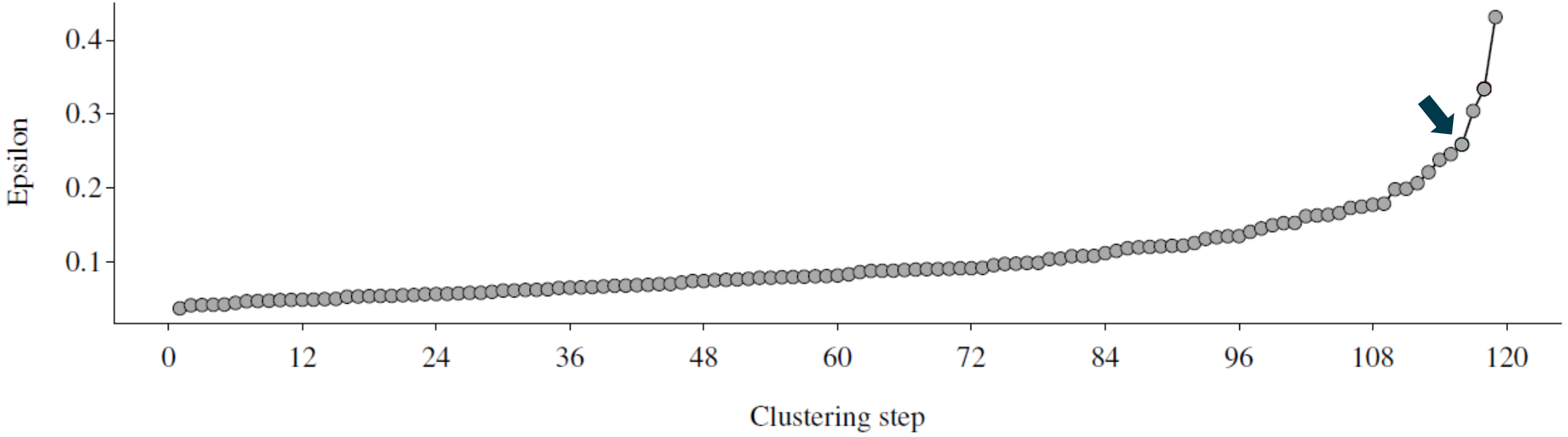
N = 120

Clustering Process (HACP)

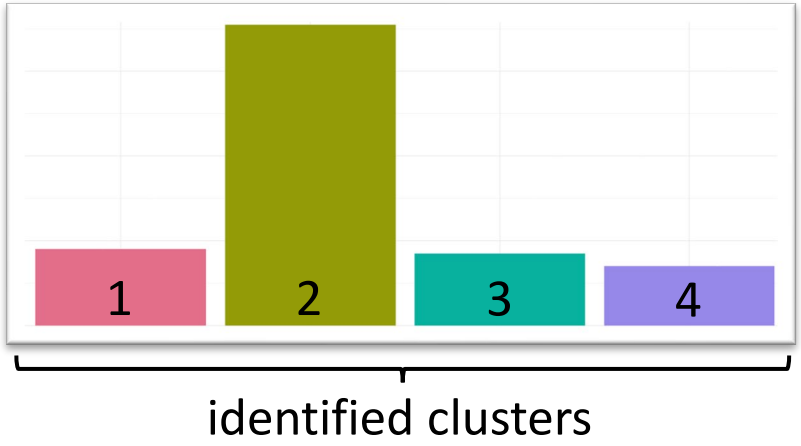


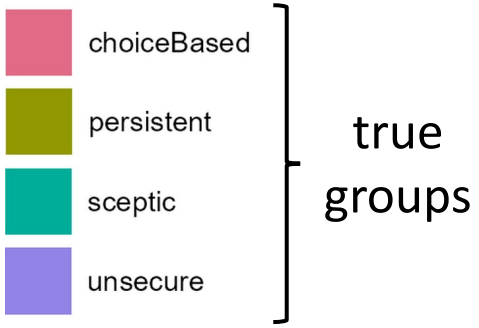
Clustering Process (HACP)

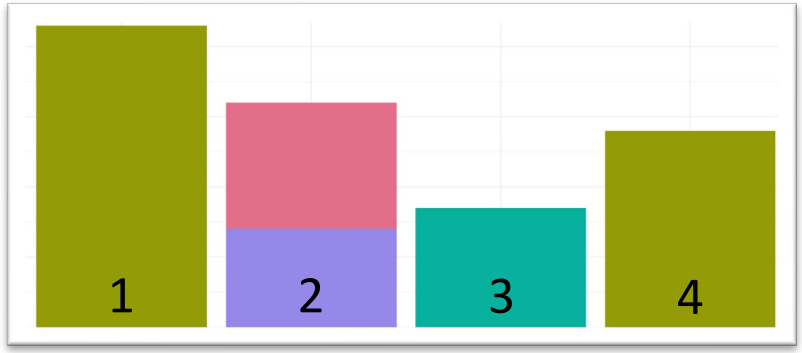


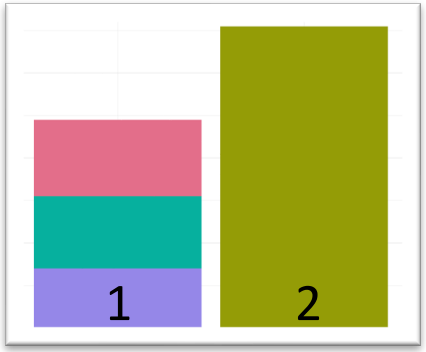


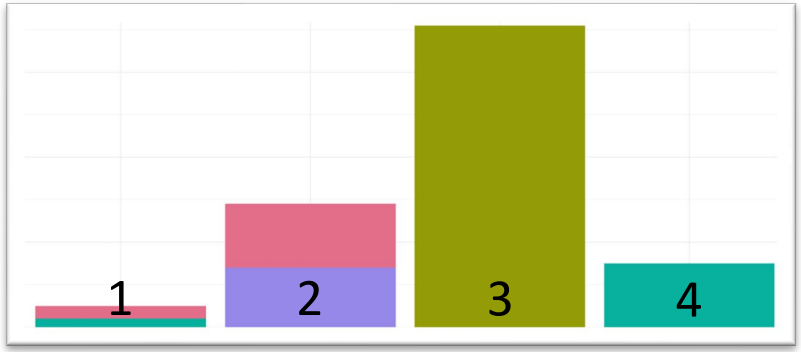
Homogeneity of the Clusters (“Fit”)

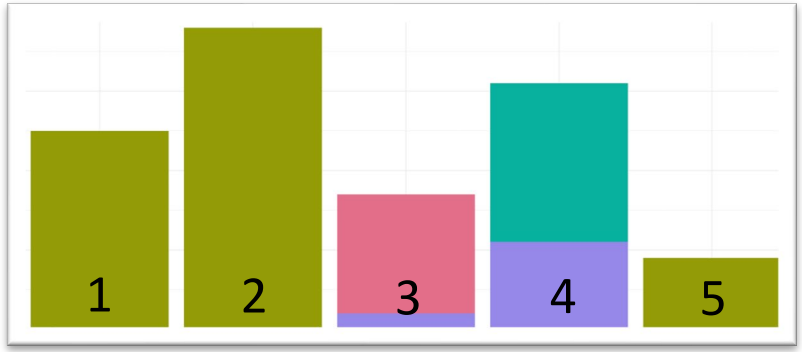




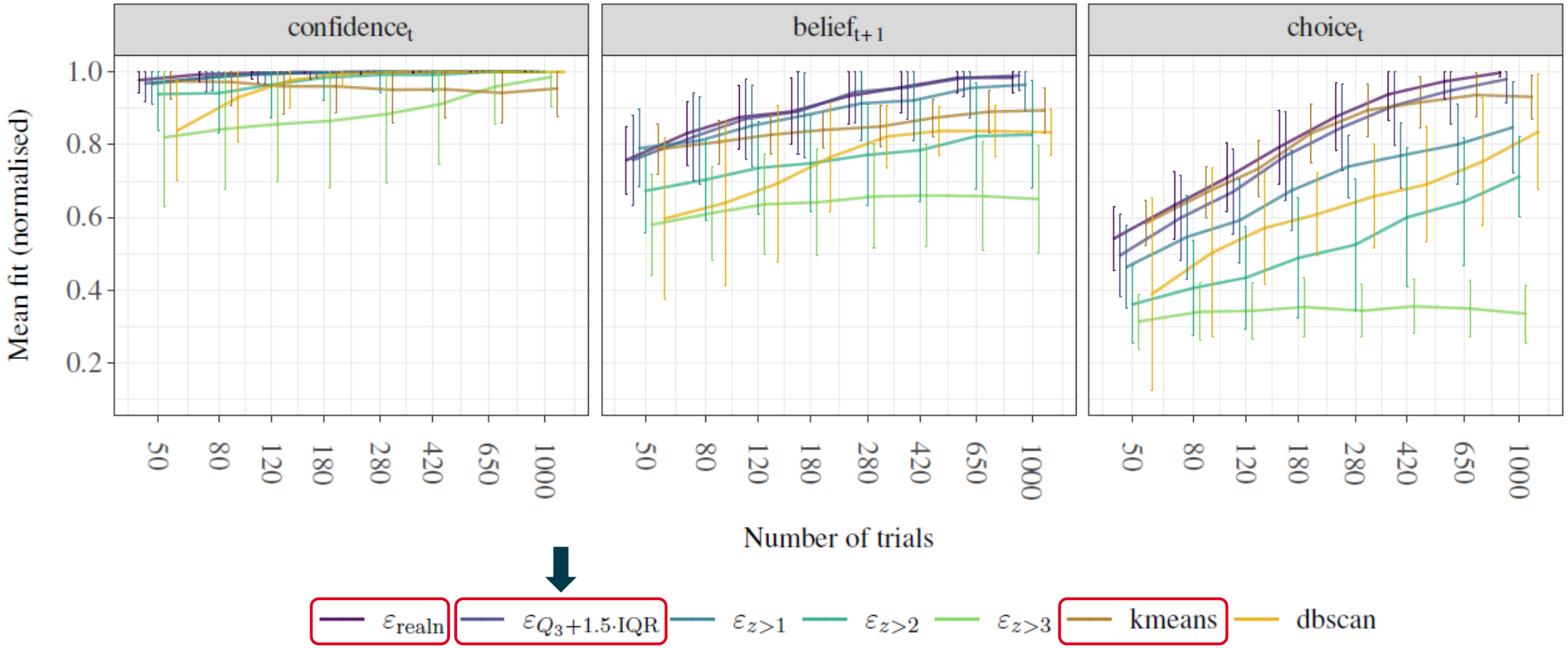




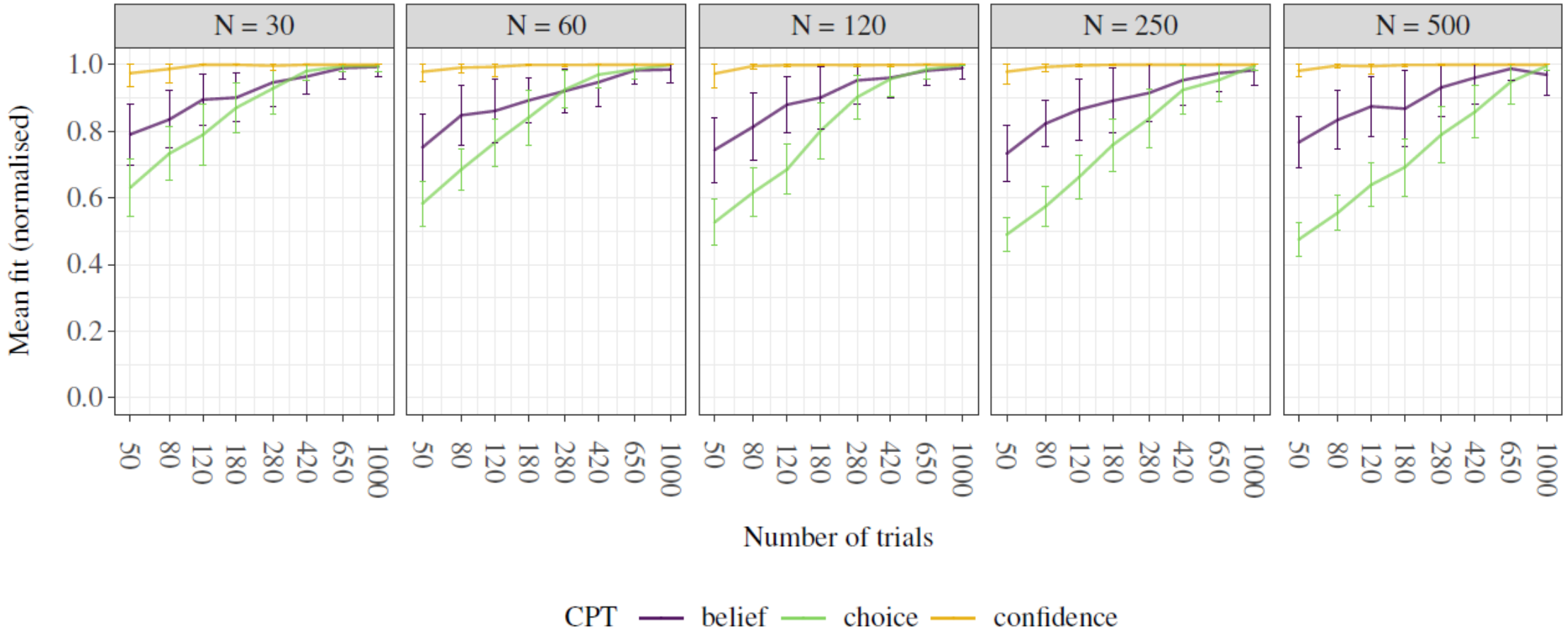




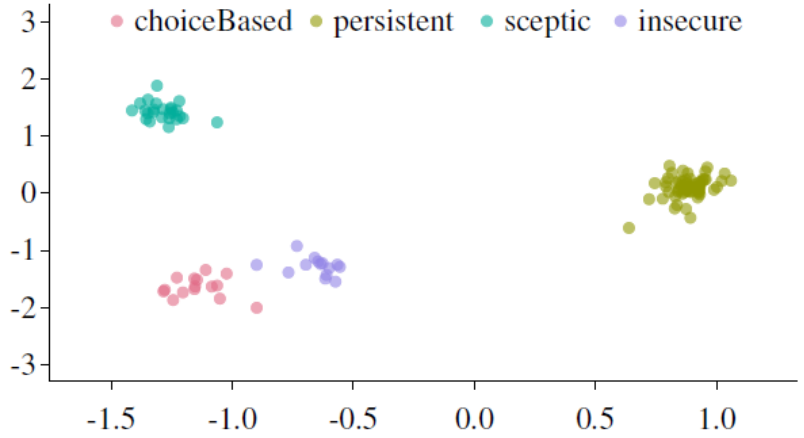
Homogeneity of the Clusters (“Fit”)



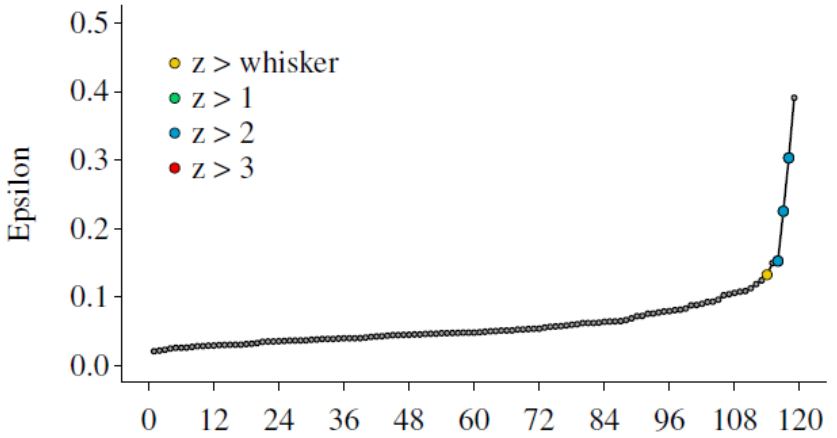
Variation: Number of Participants and Trials

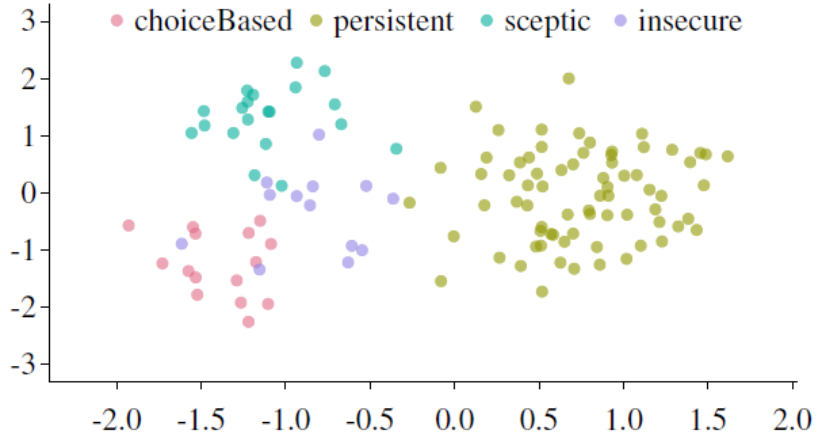


Effect of the Number of Trials

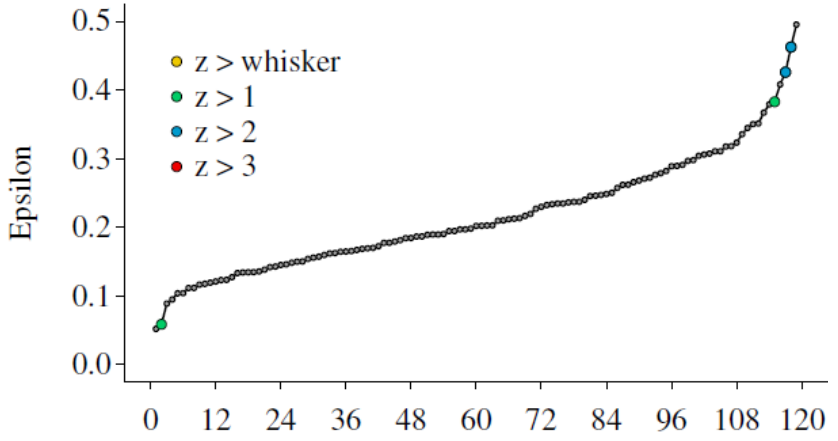


$N = 120$
with 1000 trials

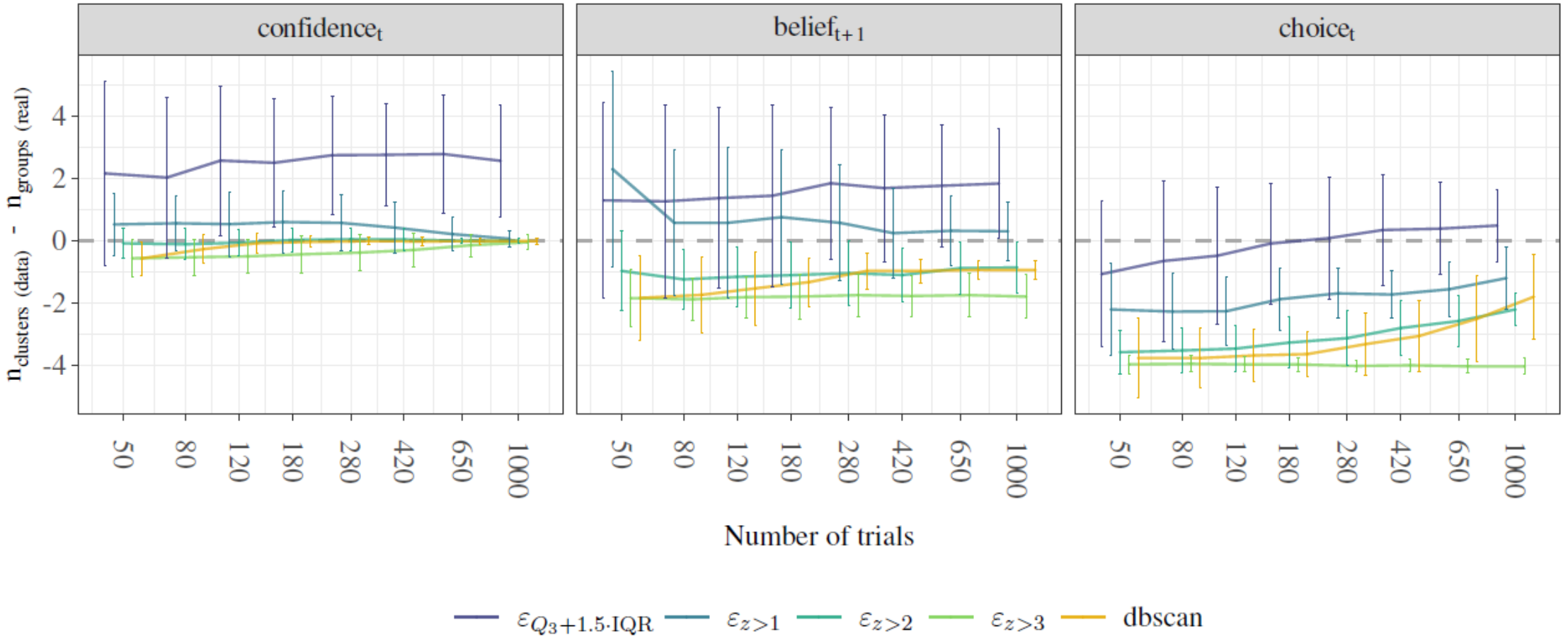




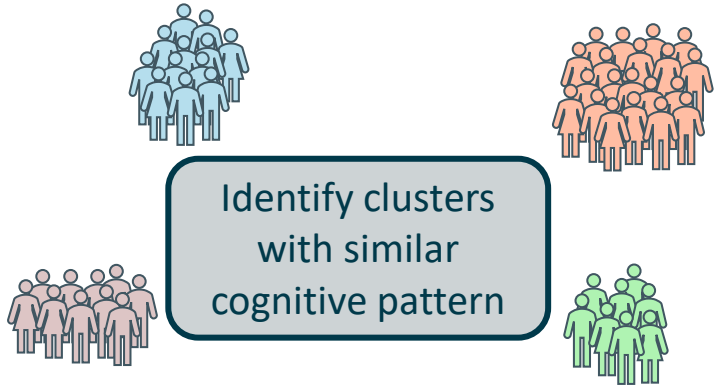
$N = 120$
with 50 trials



Number of Clusters



Conclusion: When and How?



1. Are you interested in cognitive or behavioural processes?
2. Can the process be modelled as a (dynamic) Bayesian network?
3. Do you assume individual differences *and* similarities?
4. How do you want similarity to be defined (e.g., RMSE)?
5. Do you know how many groups to expect?



No? 😮 … perfect! Make it iterative, visualise distances, and inspect the merge sequence!

6. Do you prefer more homogenous clusters or greater data aggregation?

!! Finding the correct number of groups is not trivial and depends on the data !!

⟹ For your next study: Increase the number of trials (if you can)! ☺

