Towards Learning Differentially Private Probabilistic Relational Models II

AnoMed Seminar

Malte Luttermann

German Research Center for Artificial Intelligence (DFKI)

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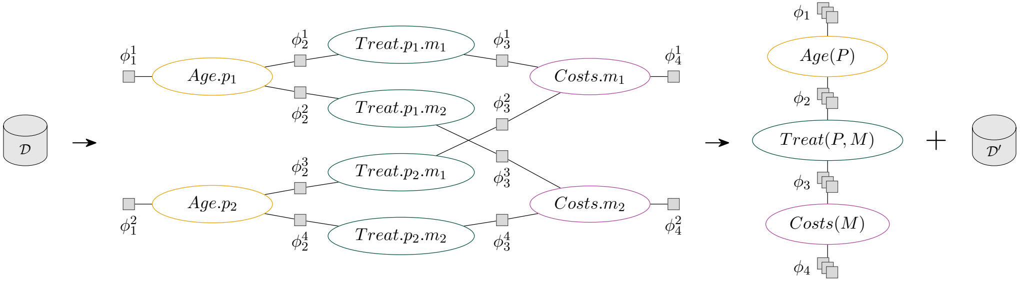
Recap – Overview

(Luttermann, Möller, and Hartwig, 2024)

Pipeline to generate synthetic relational data via probabilistic relational models:

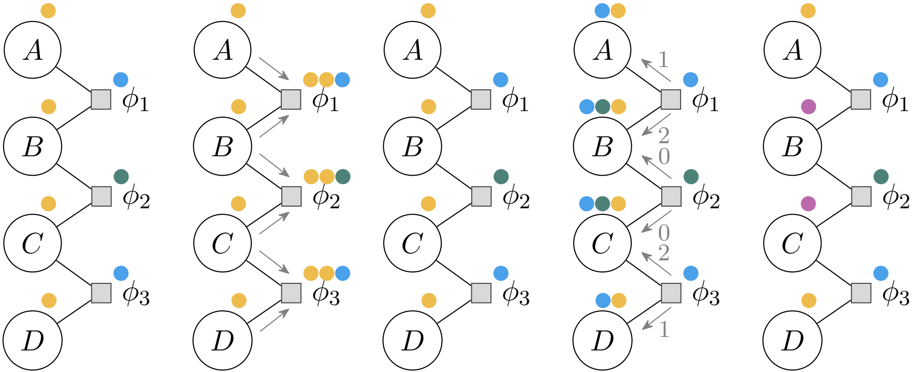
1. Construction of a factor graph
2. Transformation of the factor graph into a PRM (e.g., a parametric factor graph)
3. Sampling the PRM to generate synthetic data samples



Recap – Advanced Colour Passing Algorithm

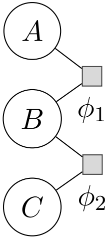
(Luttermann, Braun, et al., 2024)

- Assign colours to random variables depending on their ranges and evidence
- Assign colours to factors depending on their potentials
- Detect symmetric subgraphs by passing colours around



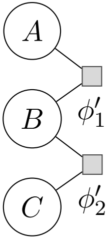
Exchangeable Factors

- Goal: Efficiently detect exchangeable factors
- Exchangeable factors encode an equivalent probability distribution



A	B	$\phi_1(A, B)$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

C	B	$\phi_2(C, B)$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4



A	B	$\phi'_1(A, B)$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

B	C	$\phi'_2(B, C)$
true	true	φ_1
true	false	φ_3
false	true	φ_2
false	false	φ_4

Detection of Exchangeable Factors

So far:

- Advanced Colour Passing algorithm to construct a lifted representation
- *Buckets* to prune the search space, then iterating over all argument permutations
- Number of iterations in the worst-case for a factor with n arguments: $O(n!)$

Detection of Exchangeable Factors

So far:

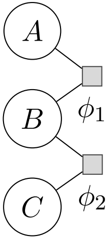
- Advanced Colour Passing algorithm to construct a lifted representation
- *Buckets* to prune the search space, then iterating over all argument permutations
- Number of iterations in the worst-case for a factor with n arguments: $O(n!)$

Next:

- Theoretical guarantees: *Buckets* to avoid iterating over permutations if possible
- Practical algorithm: Detection of Exchangeable Factors (DEFT) algorithm
- Number of iterations in the worst-case for a factor with n arguments: $O(d)$ with $d \ll n!$ in many practical settings

Buckets

- Buckets count the occurrences of specific range values in an assignment
- Each potential belongs to *exactly one* bucket
- Each bucket contains *at least one* potential



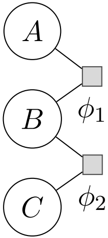
A	B	$\phi_1(A, B)$	b
true	true	φ_1	$[2, 0]$
true	false	φ_2	$[1, 1]$
false	true	φ_3	$[1, 1]$
false	false	φ_4	$[0, 2]$

B	C	$\phi_2(B, C)$	b
true	true	φ_1	$[2, 0]$
true	false	φ_3	$[1, 1]$
false	true	φ_2	$[1, 1]$
false	false	φ_4	$[0, 2]$

b	$\phi_1(b)$	$\phi_2(b)$
$[2, 0]$	$\{\varphi_1\}$	$\{\varphi_1\}$
$[1, 1]$	$\{\varphi_2, \varphi_3\}$	$\{\varphi_3, \varphi_2\}$
$[0, 2]$	$\{\varphi_4\}$	$\{\varphi_4\}$

Properties of Buckets I

- If at least one bucket is mapped to different multisets of values by ϕ_1 and ϕ_2, then ϕ_1 and ϕ_2 are not exchangeable (Luttermann, Braun, et al., 2024)



A	B	$\phi_1(A, B)$	b
true	true	φ_1	$[2, 0]$
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B	C	$\phi_2(B, C)$	b
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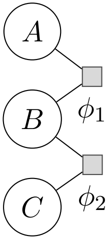
b	$\phi_1(b)$	$\phi_2(b)$
$[2, 0]$	$\{\varphi_1\}$	$\{\varphi_1\}$
$[1, 1]$	$\{\varphi_2, \varphi_3\}$	$\{\varphi_3, \varphi_2\}$
$[0, 2]$	$\{\varphi_4\}$	$\{\varphi_4\}$

Properties of Buckets II

- Two factors are exchangeable iff there exists a permutation of their arguments such that each bucket is mapped to the same ordered multiset of values



A	B	$\phi_1(A, B)$	b
true	true	φ_1	$[2, 0]$
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false	false	φ_4	$[0, 2]$

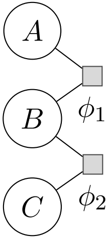
B	C	$\phi_2(B, C)$	b
true	true	φ_1	$[2, 0]$
true	false	φ_3	$[1, 1]$
false	true	φ_2	$[1, 1]$
false	false	φ_4	$[0, 2]$

b	$\phi_1^{\succ}(b)$	$\phi_2^{\succ}(b)$
$[2, 0]$	$\langle \varphi_1 \rangle$	$\langle \varphi_1 \rangle$
$[1, 1]$	$\langle \varphi_2, \varphi_3 \rangle$	$\langle \varphi_3, \varphi_2 \rangle$
$[0, 2]$	$\langle \varphi_4 \rangle$	$\langle \varphi_4 \rangle$

The Detection of Exchangeable Factors (DEFT) Algorithm

(Luttermann, Machemer, and Gehrke, 2024b)

- Iterate over buckets and search for possible rearrangements to obtain identically ordered multisets within each bucket



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Possible rearrangements:

$[2, 0]$: $B \to \{1, 2\}$, $C \to \{1, 2\}$

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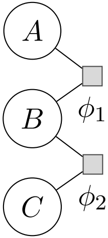

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false	false	φ_4	$[0, 2]$

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$[2, 0]$	$\langle \varphi_1 \rangle$	$\langle \varphi_1 \rangle$
$[1, 1]$	$\langle \varphi_2, \varphi_3 \rangle$	$\langle \varphi_3, \varphi_2 \rangle$
$[0, 2]$	$\langle \varphi_4 \rangle$	$\langle \varphi_4 \rangle$

Possible rearrangements:

$[2, 0]$: $B \to \{1, 2\}$, $C \to \{1, 2\}$

$[1, 1]$: $B \to \{2\}$, $C \to \{1\}$

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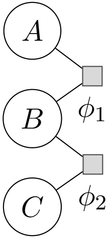

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true	false	φ_3	$[1, 1]$
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b	$\phi_1^{\succ}(b)$	$\phi_2^{\succ}(b)$
$[2, 0]$	$\langle \varphi_1 \rangle$	$\langle \varphi_1 \rangle$
$[1, 1]$	$\langle \varphi_2, \varphi_3 \rangle$	$\langle \varphi_3, \varphi_2 \rangle$
$[0, 2]$	$\langle \varphi_4 \rangle$	$\langle \varphi_4 \rangle$

Possible rearrangements:

$[2, 0]$: $B \to \{1, 2\}$, $C \to \{1, 2\}$

$[1, 1]$: $B \to \{2\}$, $C \to \{1\}$

$[0, 2]$: $B \to \{1, 2\}$, $C \to \{1, 2\}$

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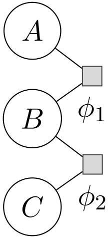
(Luttermann, Machemer, and Gehrke, 2024b)

- Iterate over buckets and search for possible rearrangements to obtain identically ordered multisets within each bucket



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true	false	φ_3	$[1, 1]$
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false	false	φ_4	$[0, 2]$

b	$\phi_1^{\succ}(b)$	$\phi_2^{\succ}(b)$
$[2, 0]$	$\langle \varphi_1 \rangle$	$\langle \varphi_1 \rangle$
$[1, 1]$	$\langle \varphi_2, \varphi_3 \rangle$	$\langle \varphi_3, \varphi_2 \rangle$
$[0, 2]$	$\langle \varphi_4 \rangle$	$\langle \varphi_4 \rangle$

Possible rearrangements:

$[2, 0]$: $B \to \{1, 2\}$, $C \to \{1, 2\}$

$[1, 1]$: $B \to \{2\}$, $C \to \{1\}$

$[0, 2]$: $B \to \{1, 2\}$, $C \to \{1, 2\}$

Intersection: $B \to \{2\}$, $C \to \{1\}$

Theoretical Guarantees of the DEFT Algorithm

- Duplicate values in buckets increase complexity
- Degree of freedom of a bucket b:

$$\mathcal{F}(b) = \prod_{\varphi \in \text{unique}(\phi^{\succ}(b))} \text{count}(\phi^{\succ}(b), \varphi)!$$

- Degree of freedom of a factor ϕ:

$$\mathcal{F}(\phi) = \min_{b \in \{b \mid b \in \mathcal{B}(\phi) \wedge |\phi^{\succ}(b)| > 1\}} \mathcal{F}(b)$$

b	$\phi_1'^{\succ}(b)$	$\phi_2'^{\succ}(b)$
$[3, 0]$	$\langle \varphi_1 \rangle$	$\langle \varphi_1 \rangle$
$[2, 1]$	$\langle \varphi_2, \varphi_3, \varphi_5 \rangle$	$\langle \varphi_3, \varphi_5, \varphi_2 \rangle$
$[1, 2]$	$\langle \varphi_4, \varphi_6, \varphi_6 \rangle$	$\langle \varphi_6, \varphi_4, \varphi_6 \rangle$
$[0, 3]$	$\langle \varphi_7 \rangle$	$\langle \varphi_7 \rangle$

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b	$\phi_1'^{\succ}(b)$	$\phi_2'^{\succ}(b)$
$[3,0]$	$\langle \varphi_1 \rangle$	$\langle \varphi_1 \rangle$
$[2,1]$	$\langle \varphi_2, \varphi_3, \varphi_5 \rangle$	$\langle \varphi_3, \varphi_5, \varphi_2 \rangle$
$[1,2]$	$\langle \varphi_4, \varphi_6, \varphi_6 \rangle$	$\langle \varphi_6, \varphi_4, \varphi_6 \rangle$
$[0,3]$	$\langle \varphi_7 \rangle$	$\langle \varphi_7 \rangle$

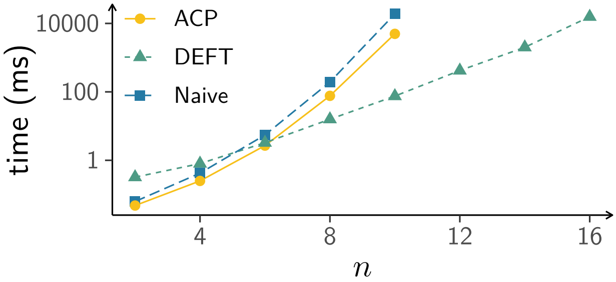
- Degree of freedom of a factor ϕ:

$$\mathcal{F}(\phi) = \min_{b \in \{b \mid b \in \mathcal{B}(\phi) \wedge |\phi^{\succ}(b)| > 1\}} \mathcal{F}(b)$$

- The number of table comparisons needed to check whether ϕ_1 and ϕ_2 are exchangeable is in $O(d)$, where $d = \min\{\mathcal{F}(\phi_1), \mathcal{F}(\phi_2)\}$ (i.e., d is factorial)
- In many practical settings it holds that $d \ll n!$
- The degree of freedom is upper-bounded by $n!$

Experiments

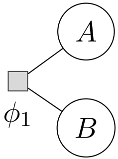
- Comparison of run times of DEFT, ACP, and a »naive« approach
- Average over exchangeable and non-exchangeable factors with a proportion of identical potentials in $\{0.0, 0.1, 0.2, 0.5, 0.8, 0.9, 1.0\}$
- Timeout after 30 minutes per instance



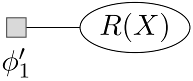
Commutative Factors

- If a factor is commutative, its neighbours might be grouped
- Use histograms to detect commutativity of factors
- Each histogram must be mapped to a unique value

A	B	$\phi_1(A, B)$
true	true	φ_1
true	false	φ_2
false	true	φ_2
false	false	φ_3



$\#_X[R(X)]$	$\phi'_1(\#_X[R(X)])$
$[2, 0]$	φ_1
$[1, 1]$	φ_2
$[0, 2]$	φ_3



Detection of Commutative Factors

So far:

- Iterate over all subsets of arguments to find commutative arguments
- Number of iterations for a factor with n arguments is in $O(2^n)$

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Next:

- Theoretical guarantees: *Buckets* to avoid iterating over all subsets
- Practical algorithm: Detection of Commutative Factors (DECOR) algorithm

Properties of Buckets I

For any subset $\boldsymbol{S}$ of commutative arguments:

- $|\boldsymbol{S}| \leq \min_{b \in \{b | b \in \mathcal{B}(\phi) \wedge |\phi(b)| > 1\}} \max_{\varphi \in \phi(b)} \text{count}(\phi(b), \varphi)$, i.e.,
- in each bucket b with $|\phi(b)| > 1$, there is a potential occurring at least $|\boldsymbol{S}|$ times

A	B	R	$\phi(A, B, R)$	b
true	true	true	φ_1	$[3, 0]$
true	true	false	φ_4	$[2, 1]$
true	false	true	φ_2	$[2, 1]$
true	false	false	φ_5	$[1, 2]$
false	true	true	φ_2	$[2, 1]$
false	true	false	φ_5	$[1, 2]$
false	false	true	φ_3	$[1, 2]$
false	false	false	φ_6	$[0, 3]$

b	$\phi(b)$
$[3, 0]$	$\langle \varphi_1 \rangle$
$[2, 1]$	$\langle \varphi_4, \varphi_2, \varphi_2 \rangle$
$[1, 2]$	$\langle \varphi_5, \varphi_5, \varphi_3 \rangle$
$[0, 3]$	$\langle \varphi_6 \rangle$

Properties of Buckets II

For a group of identical potentials in a bucket:

- Intersection of their corresponding assignments yields candidates
- E.g., for φ_2's in $[2, 1]$: $(\text{true}, \text{false}, \text{true}) \cap (\text{false}, \text{true}, \text{true}) = (\emptyset, \emptyset, \text{true})$

A	B	R	$\phi(A, B, R)$	b
true	true	true	φ_1	$[3, 0]$
true	true	false	φ_4	$[2, 1]$
true	false	true	φ_2	$[2, 1]$
true	false	false	φ_5	$[1, 2]$
false	true	true	φ_2	$[2, 1]$
false	true	false	φ_5	$[1, 2]$
false	false	true	φ_3	$[1, 2]$
false	false	false	φ_6	$[0, 3]$

b	$\phi(b)$
$[3, 0]$	$\langle \varphi_1 \rangle$
$[2, 1]$	$\langle \varphi_4, \varphi_2, \varphi_2 \rangle$
$[1, 2]$	$\langle \varphi_5, \varphi_5, \varphi_3 \rangle$
$[0, 3]$	$\langle \varphi_6 \rangle$

The Detection of Commutative Factors (DECOR) Algorithm

(Luttermann, Machemer, and Gehrke, 2024a)

- Iterate over buckets and compute candidates for commutative arguments

A	B	R	$\phi(A,B,R)$	b
true	true	true	φ_1	$[3,0]$
true	true	false	φ_4	$[2,1]$
true	false	true	φ_2	$[2,1]$
true	false	false	φ_5	$[1,2]$
false	true	true	φ_2	$[2,1]$
false	true	false	φ_5	$[1,2]$
false	false	true	φ_3	$[1,2]$
false	false	false	φ_6	$[0,3]$

b	$\phi(b)$
$[3,0]$	$\langle\varphi_1\rangle$
$[2,1]$	$\langle\varphi_4,\varphi_2,\varphi_2\rangle$
$[1,2]$	$\langle\varphi_5,\varphi_5,\varphi_3\rangle$
$[0,3]$	$\langle\varphi_6\rangle$

Initial candidates: $\{\{A,B,R\}\}$

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(Luttermann, Machemer, and Gehrke, 2024a)

- Iterate over buckets and compute candidates for commutative arguments

A	B	R	$\phi(A, B, R)$	b
true	true	true	φ_1	$[3, 0]$
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true	false	true	φ_2	$[2, 1]$
true	false	false	φ_5	$[1, 2]$
false	true	true	φ_2	$[2, 1]$
false	true	false	φ_5	$[1, 2]$
false	false	true	φ_3	$[1, 2]$
false	false	false	φ_6	$[0, 3]$

b	$\phi(b)$
$[3, 0]$	$\langle \varphi_1 \rangle$
$[2, 1]$	$\langle \varphi_4, \varphi_2, \varphi_2 \rangle$
$[1, 2]$	$\langle \varphi_5, \varphi_5, \varphi_3 \rangle$
$[0, 3]$	$\langle \varphi_6 \rangle$

Initial candidates: $\{\{A, B, R\}\}$

$[3, 0]$ $(|\phi(b)| < 2)$: $\{\{A, B, R\}\}$

The Detection of Commutative Factors (DECOR) Algorithm

(Luttermann, Machemer, and Gehrke, 2024a)

- Iterate over buckets and compute candidates for commutative arguments

A	B	R	$\phi(A, B, R)$	b
true	true	true	φ_1	$[3, 0]$
true	true	false	φ_4	$[2, 1]$
true	false	true	φ_2	$[2, 1]$
true	false	false	φ_5	$[1, 2]$
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false	false	false	φ_6	$[0, 3]$

b	$\phi(b)$
$[3, 0]$	$\langle \varphi_1 \rangle$
$[2, 1]$	$\langle \varphi_4, \varphi_2, \varphi_2 \rangle$
$[1, 2]$	$\langle \varphi_5, \varphi_5, \varphi_3 \rangle$
$[0, 3]$	$\langle \varphi_6 \rangle$

$$
\begin{aligned}
& (\text{true}, \text{false}, \text{true}) \\
\cap\ & (\text{false}, \text{true}, \text{true}) \\
=\ & (\emptyset, \quad \emptyset, \quad \text{true})
\end{aligned}
$$

Initial candidates: $\{\{A, B, R\}\}$

$[3, 0]$ $(|\phi(b)| < 2)$: $\{\{A, B, R\}\}$

$[2, 1]$: $\{\{A, B\}\}$

The Detection of Commutative Factors (DECOR) Algorithm

(Luttermann, Machemer, and Gehrke, 2024a)

- Iterate over buckets and compute candidates for commutative arguments

A	B	R	$\phi(A, B, R)$	b
true	true	true	φ_1	$[3, 0]$
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true	false	false	φ_5	$[1, 2]$
false	true	true	φ_2	$[2, 1]$
false	true	false	φ_5	$[1, 2]$
false	false	true	φ_3	$[1, 2]$
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$[3, 0]$	$\langle \varphi_1 \rangle$
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$$
\begin{aligned}
& (\text{true}, \text{false}, \text{false}) \\
\cap\ & (\text{false}, \text{true}, \text{false}) \\
=\ & (\emptyset, \quad \emptyset, \quad \text{false})
\end{aligned}
$$

Initial candidates: $\{\{A, B, R\}\}$

$[3, 0]$ $(|\phi(b)| < 2)$: $\{\{A, B, R\}\}$

$[2, 1]$: $\{\{A, B\}\}$

$[1, 2]$: $\{\{A, B\}\}$

The Detection of Commutative Factors (DECOR) Algorithm

(Luttermann, Machemer, and Gehrke, 2024a)

- Iterate over buckets and compute candidates for commutative arguments

A	B	R	$\phi(A, B, R)$	b
true	true	true	φ_1	$[3, 0]$
true	true	false	φ_4	$[2, 1]$
true	false	true	φ_2	$[2, 1]$
true	false	false	φ_5	$[1, 2]$
false	true	true	φ_2	$[2, 1]$
false	true	false	φ_5	$[1, 2]$
false	false	true	φ_3	$[1, 2]$
false	false	false	φ_6	$[0, 3]$

b	$\phi(b)$
$[3, 0]$	$\langle \varphi_1 \rangle$
$[2, 1]$	$\langle \varphi_4, \varphi_2, \varphi_2 \rangle$
$[1, 2]$	$\langle \varphi_5, \varphi_5, \varphi_3 \rangle$
$[0, 3]$	$\langle \varphi_6 \rangle$

Initial candidates:	$\{\{A, B, R\}\}$
$[3, 0]$ $(\lvert\phi(b)\rvert < 2)$:	$\{\{A, B, R\}\}$
$[2, 1]$:	$\{\{A, B\}\}$
$[1, 2]$:	$\{\{A, B\}\}$
$[0, 3]$ $(\lvert\phi(b)\rvert < 2)$:	$\{\{A, B\}\}$

Theoretical Guarantees of the DECOR Algorithm

- Avoids »naive« iteration over all 2^n subsets of arguments
- Time complexity is upper-bounded depending on the number of groups of identical potentials in the buckets

b	$\phi(b)$
$[3, 0]$	$\langle \varphi_1 \rangle$
$[2, 1]$	$\langle \varphi_4, \varphi_2, \varphi_2 \rangle$
$[1, 2]$	$\langle \varphi_5, \varphi_5, \varphi_3 \rangle$
$[0, 3]$	$\langle \varphi_6 \rangle$

Theoretical Guarantees of the DECOR Algorithm

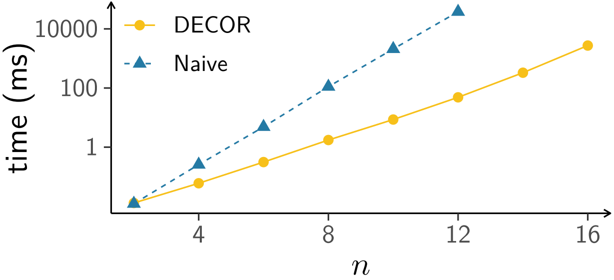
- Avoids »naive« iteration over all 2^n subsets of arguments
- Time complexity is upper-bounded depending on the number of groups of identical potentials in the buckets
- In practice, there are two possible scenarios:

b	$\phi(b)$
$[3, 0]$	$\langle \varphi_1 \rangle$
$[2, 1]$	$\langle \varphi_4, \varphi_2, \varphi_2 \rangle$
$[1, 2]$	$\langle \varphi_5, \varphi_5, \varphi_3 \rangle$
$[0, 3]$	$\langle \varphi_6 \rangle$

1. A factor contains commutative arguments
   - Potential values are likely to contain a single group of duplicates
2. A factor contains no commutative arguments
   - Potential values are likely to be distinct

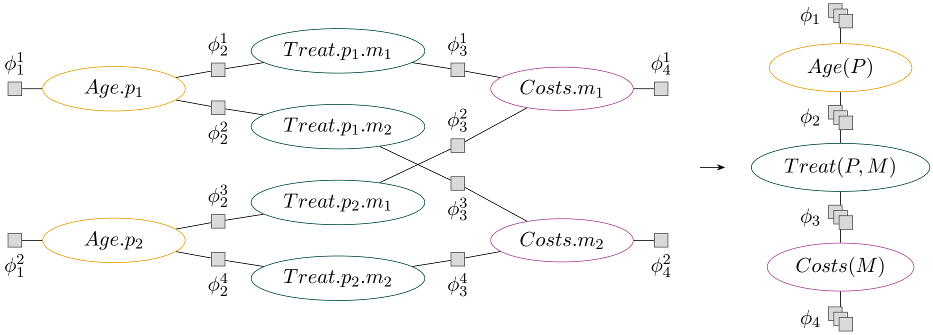
Experiments

- Comparison of run times of DECOR and the »naive« approach
- Average over factors with $k \in \{0, 2, \lfloor \frac{n}{2} \rfloor, n-1, n\}$ commutative arguments
- Timeout after five minutes per instance



Conclusion

- Problem of learning a (non-DP) PRM from a propositional factor graph solved
  - Problem of efficiently detecting exchangeable factors solved
  - Problem of efficiently detecting commutative factors solved
- Next step: DP Colour Passing Algorithm



References

Malte Luttermann, Tanya Braun, Ralf Möller, and Marcel Gehrke (2024). »Colour Passing Revisited: Lifted Model Construction with Commutative Factors«. *Proceedings of the Thirty-Eighth AAAI Conference on Artificial Intelligence (AAAI-2024)*. AAAI Press, pp. 20500–20507.

Malte Luttermann, Johann Machemer, and Marcel Gehrke (2024a). »Efficient Detection of Commutative Factors in Factor Graphs«. *Proceedings of the Twelfth International Conference on Probabilistic Graphical Models (PGM-2024)*. PMLR, pp. 38–56.

— (2024b). »Efficient Detection of Exchangeable Factors in Factor Graphs«. *Proceedings of the Thirty-Seventh International Florida Artificial Intelligence Research Society Conference (FLAIRS-2024)*. Florida Online Journals.

Malte Luttermann, Ralf Möller, and Mattis Hartwig (2024). »Towards Privacy-Preserving Relational Data Synthesis via Probabilistic Relational Models«. *Proceedings of the Forty-Seventh German Conference on Artificial Intelligence (KI-2024)*. Springer, pp. 175–189.