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Lifting Factor Graphs with Some Unknown Factors

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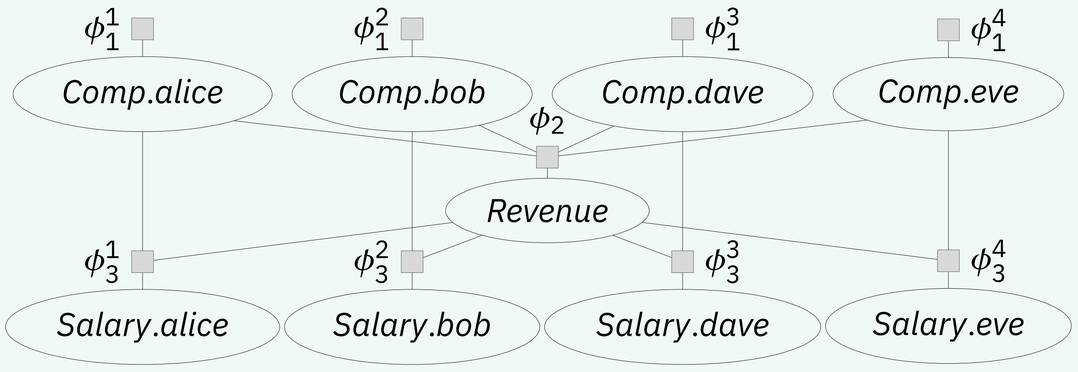
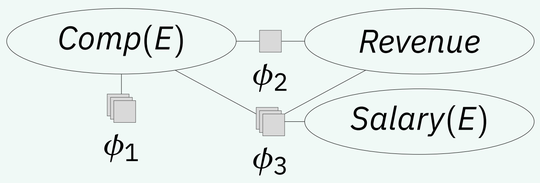
1. Motivation

- Factor graphs compactly encode a probability distribution
- Semantics of a factor graph G over a set of factors $\mathbf{\Phi}$:

$$P_G = \frac{1}{Z} \prod_{\phi \in \mathbf{\Phi}} \phi$$

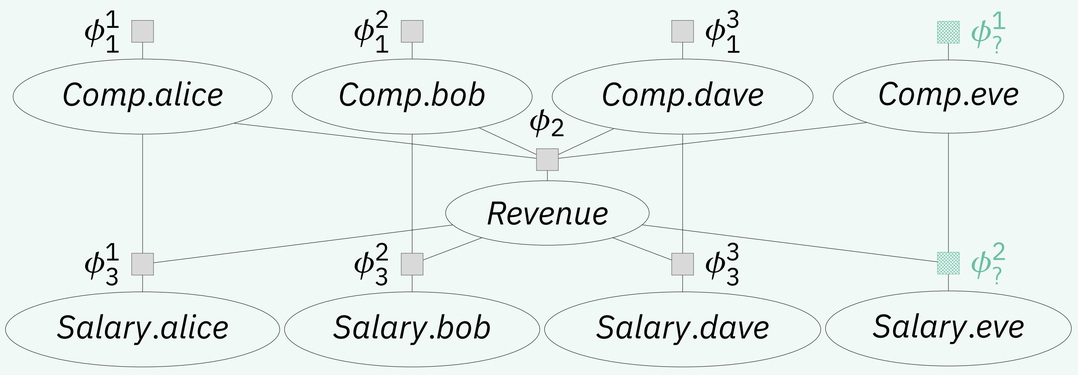


- Parametric factor graphs introduce logical variables to represent groups of random variables
- Parametric factor graphs enable lifted inference (idea: exploit indistinguishability of objects using exponentiation)



2. Problem Setup

- Goal: Lifted model construction with unknown factors
- E.g., new individual *eve* is added to the model

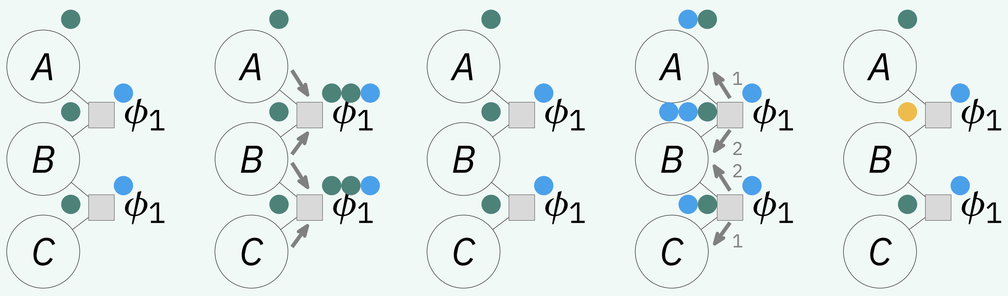


Problem Setup

Input: A factor graph that might contain unknown factors (i.e., factors with missing function definitions)
Output: A parametric factor graph

3. Prerequisite: Advanced Colour Passing (ACP)

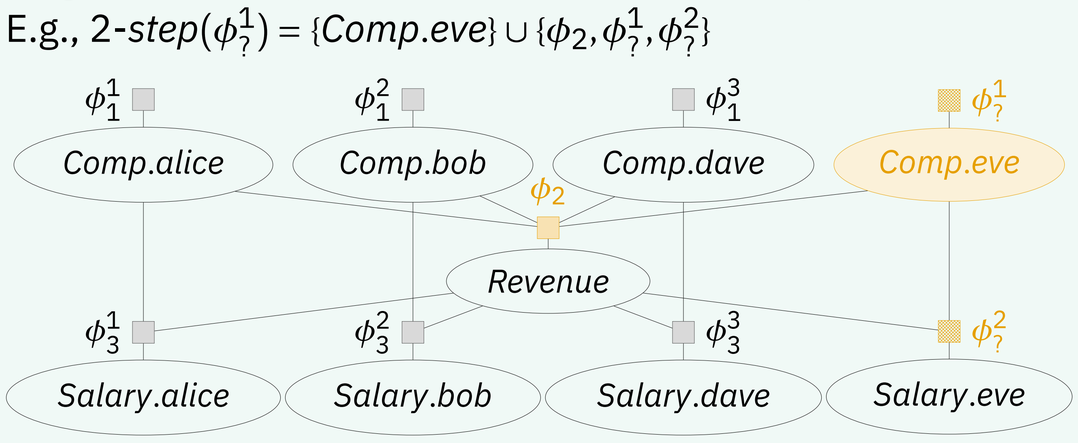
- Assign colours to random variables depending on their ranges and evidence
- Assign colour to factors depending on their potentials
- Pass colours around to detect symmetries in the graph



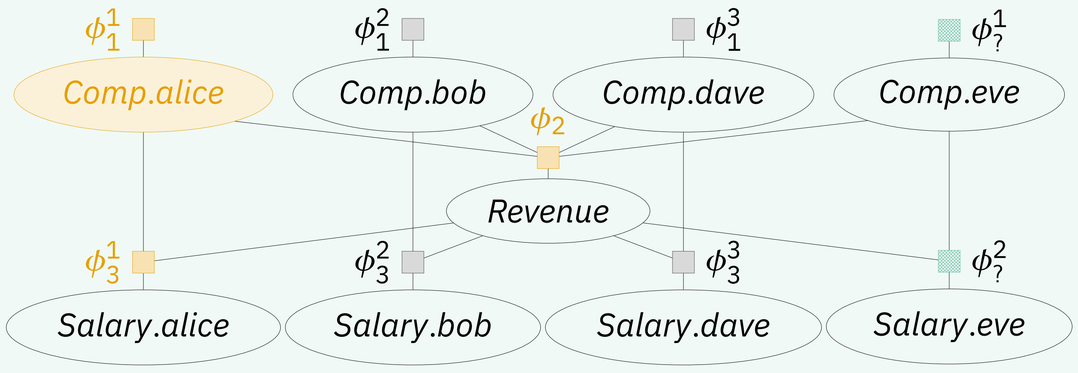
- Requires all factors to be known
- What colour to assign to the unknown factors?
  - Potentials are missing
  - Only available information: Surrounding graph structure

4. 2-Step Neighbourhoods

- The 2-step neighbourhood of a factor ϕ contains all variable nodes directly connected to ϕ and all factors that are direct neighbours of any variable node connected to ϕ
- E.g., 2-*step*$(\phi_?^1) = \{Comp.eve\} \cup \{\phi_2, \phi_?^1, \phi_?^2\}$



- ϕ_i and ϕ_j have symmetric 2-step neighbourhoods if ϕ_i and ϕ_j have the same number of neighbours and there is a one-to-one correspondence of the neighbours of ϕ_i and ϕ_j such that their observed events, ranges, and numbers of neighbours are identical
- E.g., 2-*step*$(\phi_?^1)$ is symmetric to 2-*step*(ϕ_1^1), 2-*step*(ϕ_1^2), 2-*step*(ϕ_1^3)



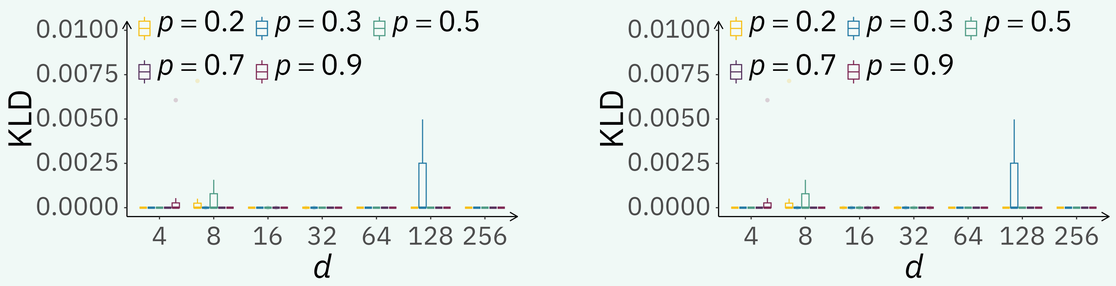
- Potential tables from known factors can be transferred to unknown factors

5. The Lifting Factor Graphs with Some Unknown Factors (LIFAGU) Algorithm

(1) Known factors are coloured according to their potential table
(2) Unknown factors are coloured according to their 2-step neighbourhood
(3) Assign unknown factors and known factors the same colour if their 2-step neighbourhoods are symmetric
(4) Transfer potential tables from known factors to unknown factors
(5) Run the ACP algorithm using the previously assigned colours

6. Experiments

- Generate factor graphs where all factors are known
- Randomly remove potentials of 5-10 percent of the factors
- Run LIFAGU to obtain a lifted representation
- Perform probabilistic inference on the ground truth and the lifted representation and measure Kullback-Leibler divergence (KLD)



- Left: 5% unknown factors, right: 10% unknown factors
- One cohort contains a proportion of p of all random variables whereas the other cohorts share the remaining proportion of $1 - p$ of the random variables uniformly at random