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Approximate Lifted Model Construction

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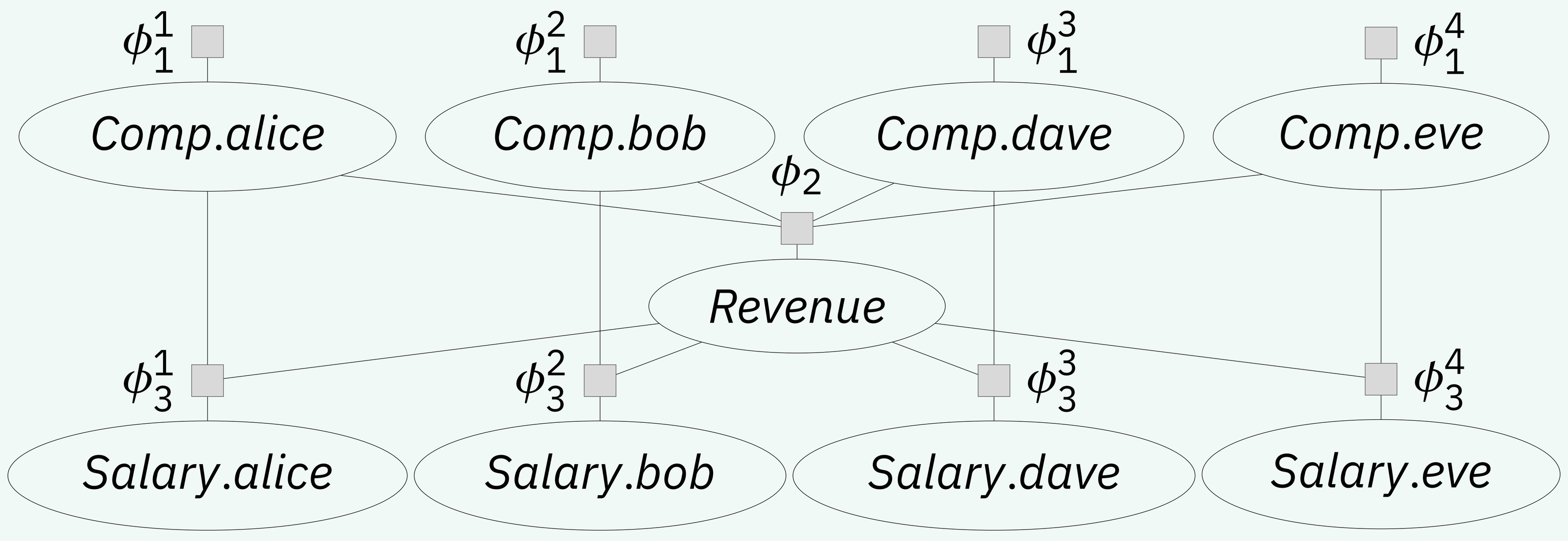
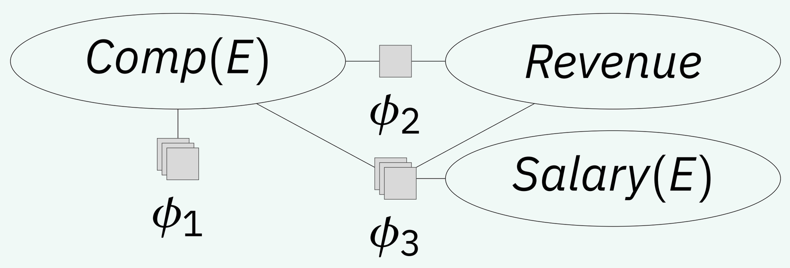
1. Motivation and Problem Setup

- Factor graphs compactly encode a probability distribution
- Semantics of a factor graph G over a set of factors $\boldsymbol{\Phi}$:

$$P_G = \frac{1}{Z} \prod_{\phi \in \boldsymbol{\Phi}} \phi$$



- Parametric factor graphs introduce logical variables to represent groups of random variables
- Parametric factor graphs enable lifted inference (idea: exploit indistinguishability of objects using exponentiation)



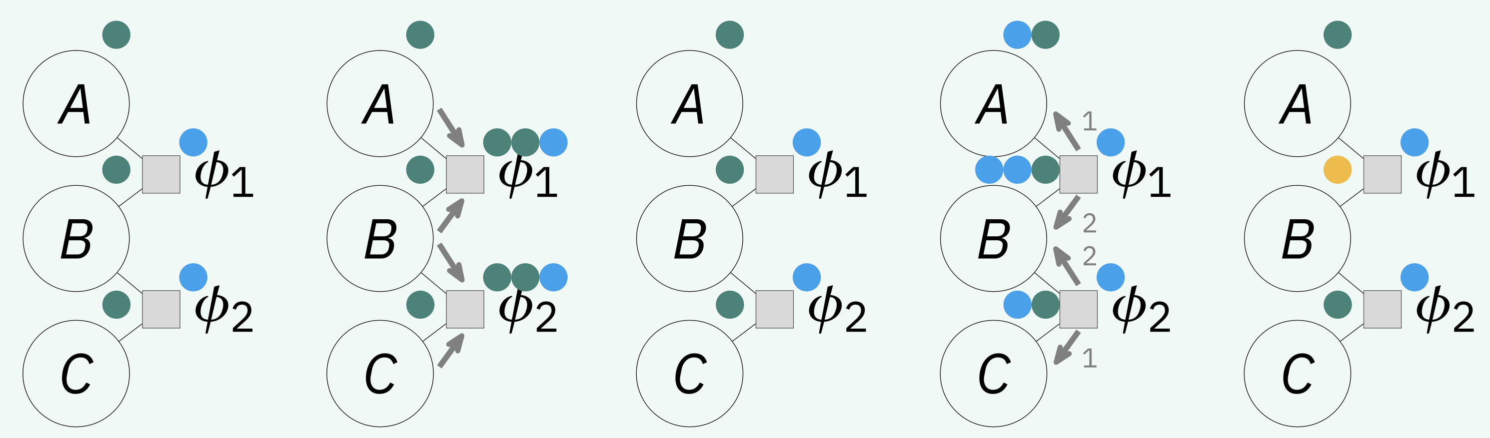
Problem Setup

Input: A factor graph G

Output: A parametric factor graph entailing approximately equivalent semantics as G
- With a minimal approximation error
- With theoretical guarantees for query results

2. Previous Work: Advanced Colour Passing (ACP)

- Assign colours to random variables according to their ranges and evidence
- Assign colours to factors according to their potential tables
- Pass colours around to detect symmetries in the graph



- Limitation: Potentials of factors must be strictly equal
  - Solution: ε-equivalence instead of strict equivalence

3. ε-Equivalence

- Potentials $\varphi_1 \in \mathbb{R}^+$ and $\varphi_2 \in \mathbb{R}^+$ are ε-equivalent if

$$\varphi_1 \in [\varphi_2 \cdot (1-\varepsilon), \varphi_2 \cdot (1+\varepsilon)] \text{ and } \varphi_2 \in [\varphi_1 \cdot (1-\varepsilon), \varphi_1 \cdot (1+\varepsilon)]$$

- Factors ϕ_1 and ϕ_2 are ε-equivalent if all potentials in their potential tables are pairwise ε-equivalent
- For example, ϕ_1 and ϕ_2 are ε-equivalent for $\varepsilon = 0.1$:

A	B	$\phi_1(A,B)$
true	true	0.81
true	false	0.32
false	true	0.51
false	false	0.21

C	B	$\phi_2(C,B)$
true	true	0.84
true	false	0.31
false	true	0.51
false	false	0.20

- Challenge: To exploit exponentiation in lifted inference, potential tables of factors in a group must be identical

4. The ε-Advanced Colour Passing (ε-ACP) Algorithm

ε-ACP is a generalisation of ACP that proceeds in three phases:

(i) Compute groups of pairwise ε-equivalent factors

(ii) Assign colours to factors according to the previously computed groups and run the colour passing procedure from ACP

(iii) Ensure identical potentials in resulting groups of factors
- Goal: Apply smallest possible change to potential tables
- Formally: Given a group of pairwise ε-equivalent factors $\boldsymbol{G} = \{\phi_1, \dots, \phi_k\}$, compute a single factor ϕ^* that fulfils

$$\phi^* = \arg\min_{\phi_j} \sum_{\phi_i \in \boldsymbol{G}} Err(\phi_i, \phi_j),$$

where $Err(\phi_i, \phi_j)$ is the sum of squared deviations between the potentials of ϕ_i and ϕ_j:

$$Err(\phi_i, \phi_j) = \sum_{(r_1,\dots,r_n)} \Big(\phi_i(r_1,\dots,r_n) - \phi_j(r_1,\dots,r_n) \Big)^2$$

- Ensure identical potentials by replacing all factors in $\boldsymbol{G}$ by ϕ^*

- Theorem: The optimal choice for ϕ^* is the arithmetic mean

$$\phi^*(r_1,\dots,r_n) = \frac{1}{k} \sum_{i=1}^{k} \phi_i(r_1,\dots,r_n).$$

- Corollary: If $\varepsilon = 0$, ε-ACP is equivalent to ACP.

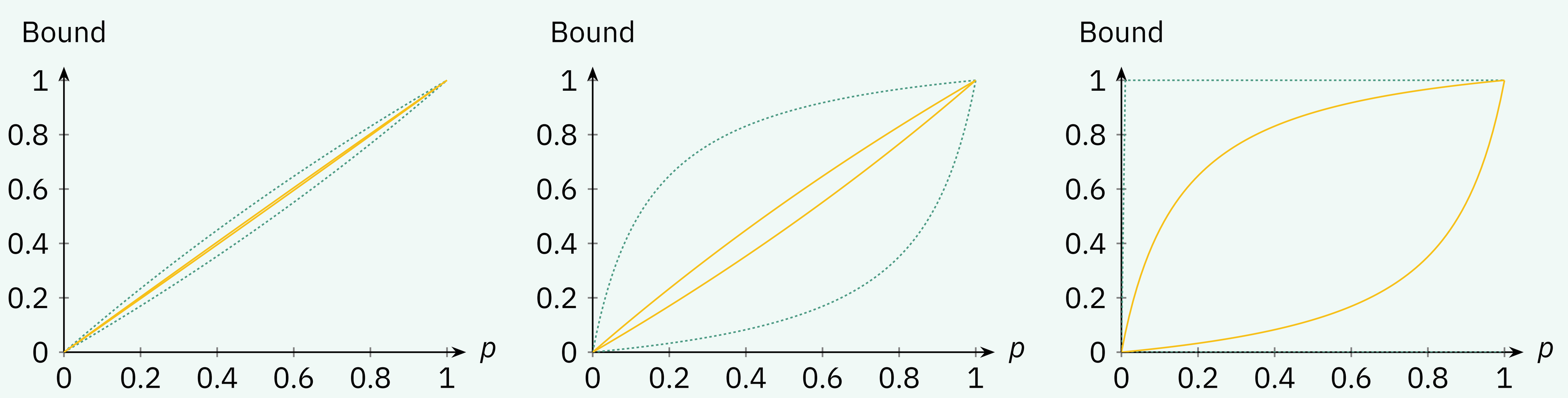
5. Bounding the Change in Query Results

- Theorem: With $p = P_M(R = r \mid \boldsymbol{e})$ being a query in any factor graph M and $p' = P_{M'}(R = r \mid \boldsymbol{e})$ being a query in the output M' of ε-ACP when run on M, the change in the query result is bounded by

$$\frac{pe^{-\delta}}{p(e^{-\delta}-1)+1} \le p' \le \frac{pe^{\delta}}{p(e^{\delta}-1)+1},$$

where $\delta \le \big[\ln{(1+\varepsilon)^m}\big] - \big[\ln{(1-\varepsilon)^m}\big]$ with m being the number of factors in M.

- Graphical illustration of the bound for $m = 10$ (left), $m = 100$ (middle), and $m = 1000$ (right) factors:



- x-axes: Original probability p, y-axes: Bound on the change in p
- Dashed (green) line: $\varepsilon = 0.01$, solid (yellow) line: $\varepsilon = 0.001$
- Theorem: The optimal bound for δ is given by

$$\delta \le \ln \left(\frac{(1 + \frac{m-1}{m}\varepsilon)(1+\varepsilon)}{1 + \frac{1}{m}\varepsilon} \right)^m.$$

- Bounds apply to arbitrary queries and factor graphs

6. Experiments

- Investigate the trade-off between exactness and compactness
- Left: Comparison of run times for lifted probabilistic inference
- Right: Quotients of query results p' in the resulting parametric factor graph and their ground truth p in the original factor graph