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Efficient Detection of Commutative Factors in Factor Graphs

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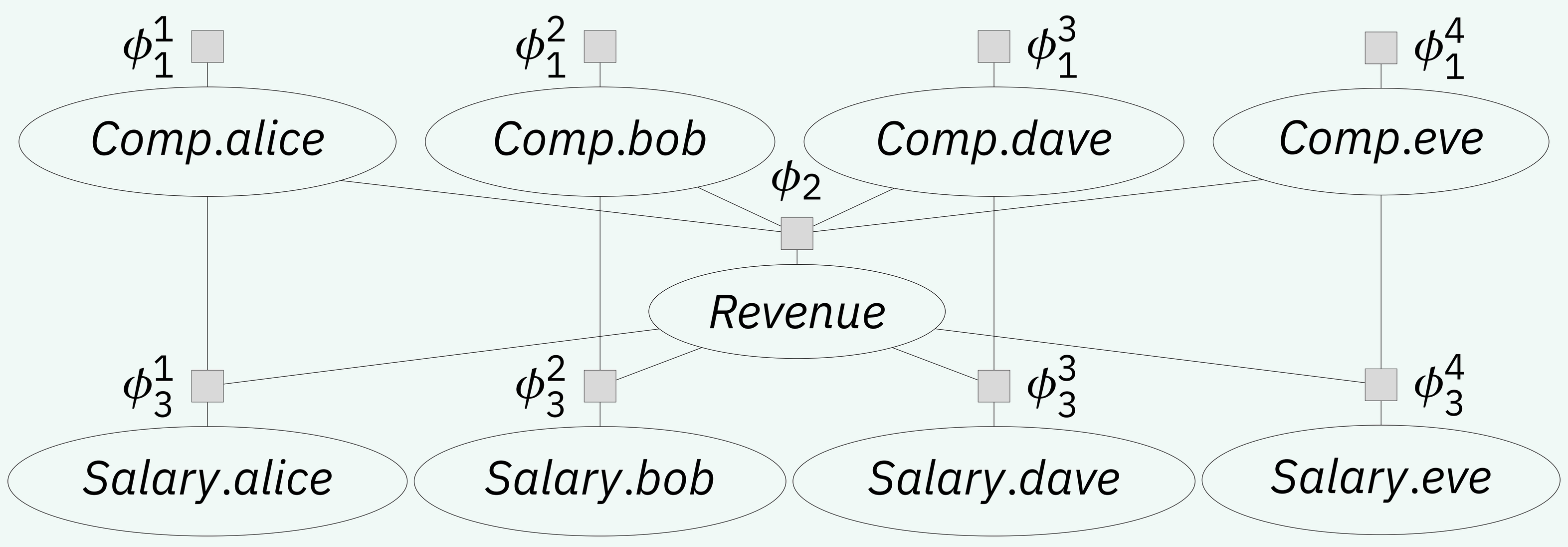
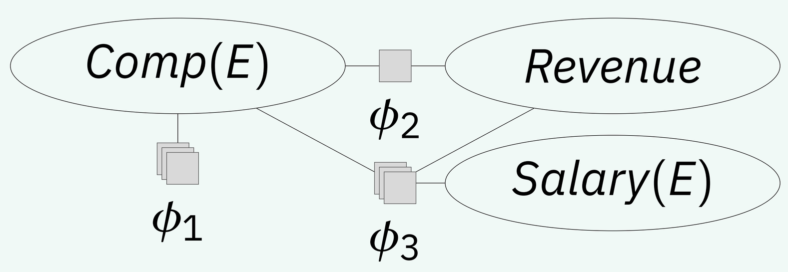
1. Motivation

- Factor graphs compactly encode a probability distribution
- Semantics of a factor graph G over a set of factors $\boldsymbol{\Phi}$:

$$P_G = \frac{1}{Z} \prod_{\phi \in \boldsymbol{\Phi}} \phi$$

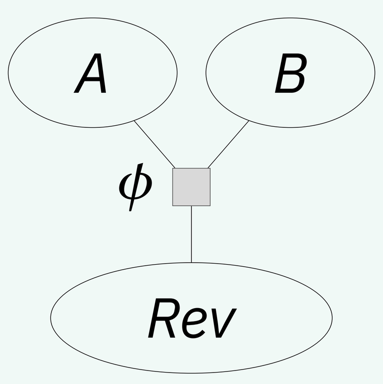


- Parametric factor graphs introduce logical variables to represent groups of random variables
- Parametric factor graphs enable lifted inference (idea: exploit indistinguishability of objects using exponentiation)



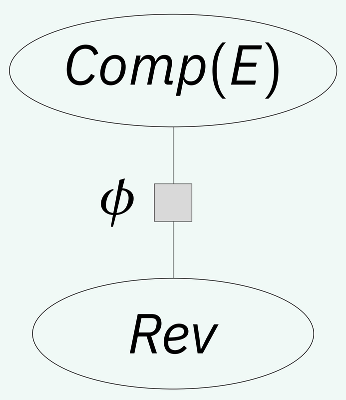
2. Problem Setup

- Goal: Efficiently detect commutative factors
- Commutative factors can be compressed



A	B	Rev	$\phi(A,B,Rev)$
true	true	true	φ_1
true	true	false	φ_4
true	false	true	φ_2
true	false	false	φ_5
false	true	true	φ_2
false	true	false	φ_5
false	false	true	φ_3
false	false	false	φ_6

↓



$\#_E[Comp(E)]$	Rev	$\phi(\#_E[Comp(E)],Rev)$
[2,0]	true	φ_1
[1,1]	true	φ_2
[0,2]	true	φ_3
[2,0]	false	φ_4
[1,1]	false	φ_5
[0,2]	false	φ_6

3. Buckets

- Buckets count occurrences of range values in an assignment

A	B	R	$\phi(A,B,R)$	b
true	true	true	φ_1	[3,0]
true	true	false	φ_4	[2,1]
true	false	true	φ_2	[2,1]
true	false	false	φ_5	[1,2]
false	true	true	φ_2	[2,1]
false	true	false	φ_5	[1,2]
false	false	true	φ_3	[1,2]
false	false	false	φ_6	[0,3]

b	$\phi(b)$
[3,0]	$\langle\varphi_1\rangle$
[2,1]	$\langle\varphi_4,\varphi_2,\varphi_2\rangle$
[1,2]	$\langle\varphi_5,\varphi_5,\varphi_3\rangle$
[0,3]	$\langle\varphi_6\rangle$

4. Detection of Commutative Factors (DECOR) Algorithm

Properties of buckets:

- For any subset $\boldsymbol{S}$ of commutative arguments:

$$|\boldsymbol{S}| \leq \min_{b \in \{b \mid b \in \mathscr{B}(\phi) \wedge |\phi(b)|>1\}} \max_{\varphi \in \phi(b)} \operatorname{count}(\phi(b),\varphi)$$

  (in each bucket b with $|\phi(b)| > 1$, there is a potential occurring at least $|\boldsymbol{S}|$ times)
- For each group of identical potentials in a bucket, the intersection of their corresponding assignments yields candidates for commutative arguments—e.g., for φ_2's in $[2,1]$:

$$(\mathrm{true},\mathrm{false},\mathrm{true}) \cap (\mathrm{false},\mathrm{true},\mathrm{true}) = (\varnothing,\varnothing,\mathrm{true})$$

A	B	R	$\phi(A,B,R)$	b
true	true	true	φ_1	[3,0]
true	true	false	φ_4	[2,1]
true	false	true	φ_2	[2,1]
true	false	false	φ_5	[1,2]
false	true	true	φ_2	[2,1]
false	true	false	φ_5	[1,2]
false	false	true	φ_3	[1,2]
false	false	false	φ_6	[0,3]

b	$\phi(b)$
[3,0]	$\langle\varphi_1\rangle$
[2,1]	$\langle\varphi_4,\varphi_2,\varphi_2\rangle$
[1,2]	$\langle\varphi_5,\varphi_5,\varphi_3\rangle$
[0,3]	$\langle\varphi_6\rangle$

DECOR iterates over buckets and computes candidates for commutative arguments:

- Initial candidates: $\{\{A,B,R\}\}$
- Candidates for bucket $[3,0]$: Skipped because $|\phi(b)| < 2$
- Candidates for bucket $[2,1]$: $\{\{A,B\}\}$

$$(\mathrm{true},\mathrm{false},\mathrm{true}) \cap (\mathrm{false},\mathrm{true},\mathrm{true}) = (\varnothing,\varnothing,\mathrm{true})$$

- Candidates for bucket $[1,2]$: $\{\{A,B\}\}$

$$(\mathrm{true},\mathrm{false},\mathrm{false}) \cap (\mathrm{false},\mathrm{true},\mathrm{false}) = (\varnothing,\varnothing,\mathrm{false})$$

- Candidates for bucket $[0,3]$: Skipped because $|\phi(b)| < 2$
- Result (intersection over all buckets): $\{\{A,B\}\}$

5. Theoretical Guarantees

Given a factor $\phi(R_1,\ldots,R_n)$:

- DECOR avoids »naive« iteration over all 2^n subsets of arguments
- Time complexity is upper-bounded depending on the number of groups of identical potentials in the buckets
- Expected worst-case time complexity of DECOR is in

$$O\left(|\mathscr{B}(\phi)| \cdot k \cdot \binom{n}{n/2} + r^n \cdot n\right),$$

  where k is the maximum number of potentials within a bucket entailed by ϕ and r is the number of range values of $R_1,\ldots,R_n$

6. Experiments

- Comparison of run times of DECOR and the »naive« approach
- Average run times over factors with $k \in \{0, 2, \lfloor\frac{n}{2}\rfloor, n-1, n\}$ commutative arguments
- Timeout after five minutes per instance