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Colour Passing Revisited: Lifted Model Construction with Commutative Factors

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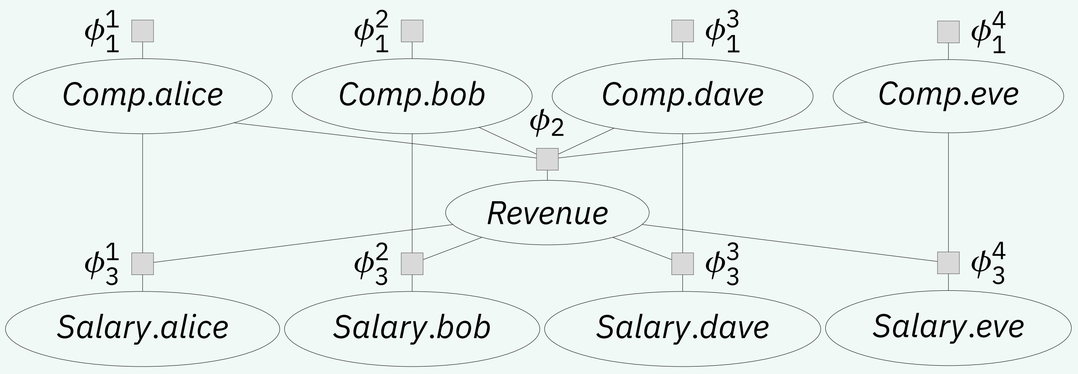
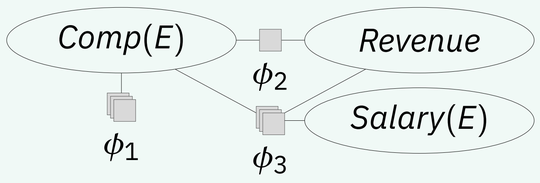
1. Motivation and Problem Setup

- Factor graphs compactly encode a probability distribution
- Semantics of a factor graph G over a set of factors $\mathbf{\Phi}$:

$$P_G = \frac{1}{Z} \prod_{\phi \in \mathbf{\Phi}} \phi$$



- Parametric factor graphs introduce logical variables to represent groups of random variables



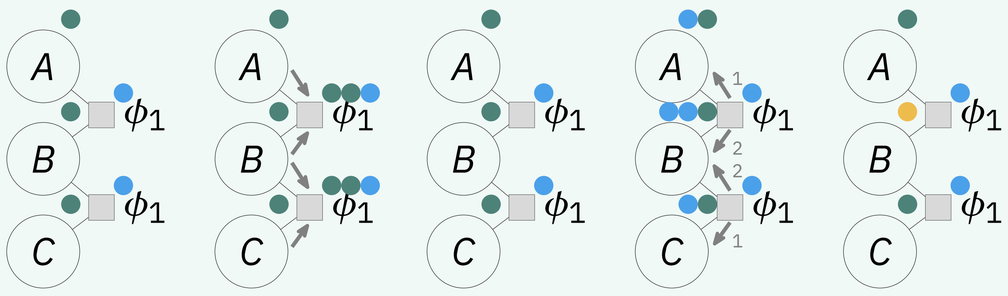
Problem Setup

Input: A factor graph G
Output: A parametric factor graph entailing equivalent semantics as G

- Under consideration of commutative factors
- Independent of the order of factor's arguments

2. Previous Work: Colour Passing (CP) Algorithm

- Assign colours to random variables depending on their ranges and evidence
- Assign colour to factors depending on their potentials
- Pass colours around to detect symmetries in the graph

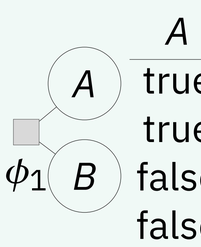


Limitations:

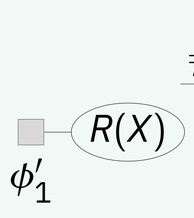
(L1) Commutative factors are not recognised
(L2) Requires fixed argument orders to detect factor equivalence
→ (L1)+(L2) result in missed graph symmetries
(L3) No logical variables are introduced
→ (L3) bounds CP to a specific inference algorithm

3. Commutative Factors (L1)

- Histograms: Efficiently detect commutative factors
- Histograms: Compactly represent commutative factors
- Also applicable to a subset of arguments



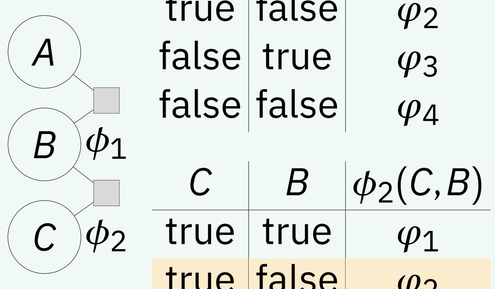
A	B	$\phi_1(A,B)$
true	true	φ_1
true	false	φ_2
false	true	φ_2
false	false	φ_3



$\#_X[R(X)]$	$\phi_1'(\#_X[R(X)])$
[2,0]	φ_1
[1,1]	φ_2
[0,2]	φ_3

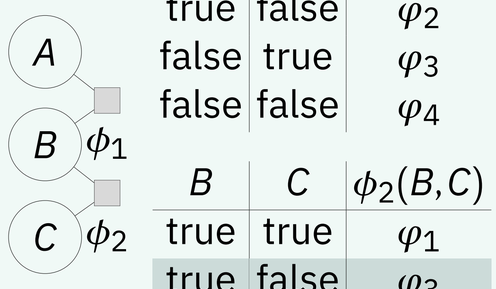
4. Permuted Factors (L2)

- Histograms: Efficient filter condition to check factor equivalence
- Histograms: Restrict possible permutations of factor arguments



A	B	$\phi_1(A,B)$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

C	B	$\phi_2(C,B)$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

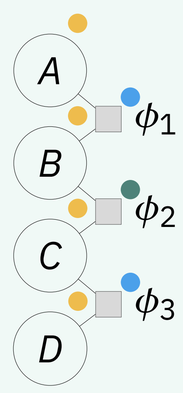


A	B	$\phi_1(A,B)$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

B	C	$\phi_2(B,C)$
true	true	φ_1
true	false	φ_3
false	true	φ_2
false	false	φ_4

5. The Advanced Colour Passing (ACP) Algorithm (L1-L3)

Input:

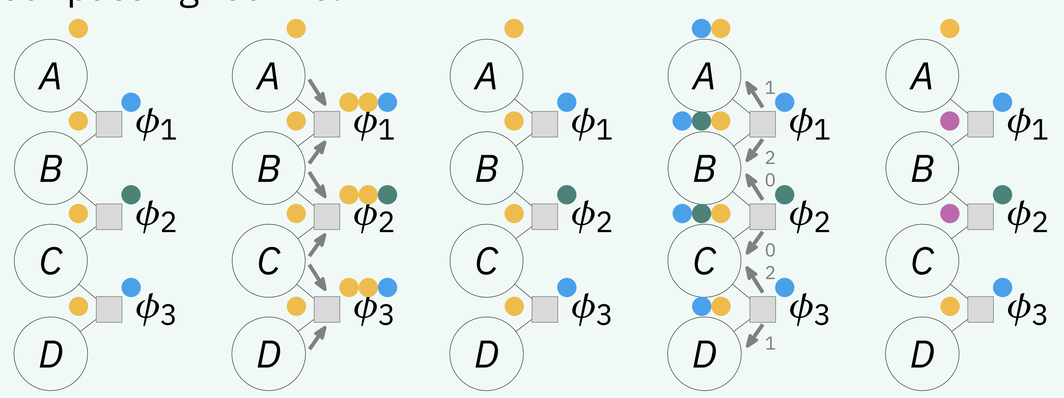


A	B	$\phi_1(A,B)$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

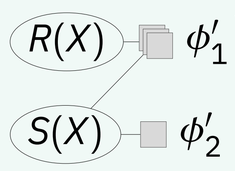
$C \leftrightarrow D$		$\phi_3(C,D)$
true	true	φ_1
true	false	φ_3
false	true	φ_2
false	false	φ_4

B	C	$\phi_2(B,C)$
true	true	φ_5
true	false	φ_6
false	true	φ_6
false	false	φ_7

Colour passing routine:



Output:



$R(X)$	$S(X)$	$\phi_1'(R(X),S(X))$
true	true	φ_1
true	false	φ_2
false	true	φ_3
false	false	φ_4

$\#_X[S(X)]$	$\phi_2'(\#_X[S(X)])$
[2,0]	φ_5
[1,1]	φ_6
[0,2]	φ_7

- Introduction of logical variables for domain-liftable models (L3)

6. Experiments

- Comparison of run times for lifted probabilistic inference
- Left: Factor graphs with 1 commutative factor (no permuted argument orders)
- Right: Factor graphs where the arguments of 3 percent of the factors are permuted (no commutative factors)